

Why is reliability index less than or equal to 1?

According to Classical Test Theory,

$$X_i = T_i + E_i$$

Find the variance of  $X_i$  from variance of  $T_i$  and  $E_i$

$$\sigma_{X_i}^2 = \sigma_{T_i + E_i}^2$$

$$\sigma_{X_i}^2 = \sigma_{T_i}^2 + \sigma_{E_i}^2 + 2\sigma_{T_i E_i} \text{ ————— (1)}$$

From the assumption of independence between true and error score

$$\rho_{T_i E_i} = 0$$

$$\frac{\sigma_{T_i E_i}}{\sigma_{T_i} \sigma_{E_i}} = 0$$

This equation is true when

$$\sigma_{T_i E_i} = 0$$

Plug in Equation 1

$$\sigma_{X_i}^2 = \sigma_{T_i}^2 + \sigma_{E_i}^2 + 2(0)$$

$$\sigma_{X_i}^2 = \sigma_{T_i}^2 + \sigma_{E_i}^2$$

$$\frac{\sigma_{T_i}^2}{\sigma_{X_i}^2} = 1 - \frac{\sigma_{E_i}^2}{\sigma_{X_i}^2}$$

$$\rho_{T_i X_i} = 1 - \frac{\sigma_{E_i}^2}{\sigma_{X_i}^2}$$

Because  $\sigma_{E_i}^2$  is always greater than or equal to 0 (property of variance),  $1 - \frac{\sigma_{E_i}^2}{\sigma_{X_i}^2}$  or

$\rho_{T_i X_i}$  is always less than or equal to 1.