

Sample size for desired accuracy in Cohen's d estimation in multilevel designs

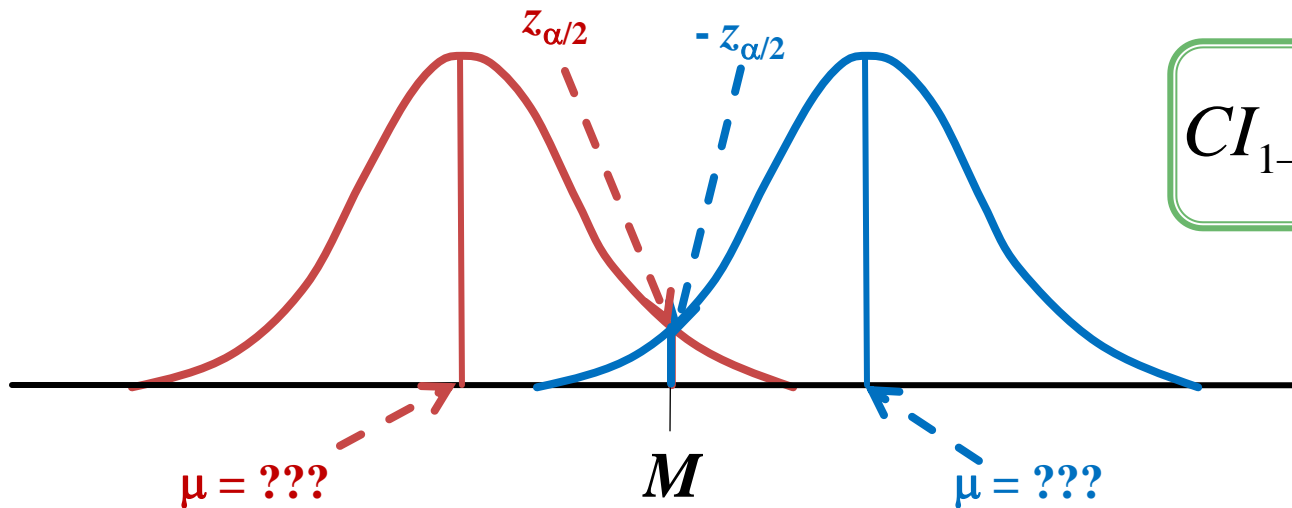
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Introduction

- Significance Testing Problems
- Advantages of Effect Size
- Confidence Interval of Effect Size

Confidence Interval of Effect Size

- Example → One sample z-test
 - Confidence interval of population mean



$$CI_{1-\alpha} = M \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$z_{\alpha/2} = \frac{M - \mu}{\frac{\sigma}{\sqrt{n}}}; \mu = z_{\alpha/2} \frac{\sigma}{\sqrt{n}} + M$$

$$-z_{\alpha/2} = \frac{M - \mu}{\frac{\sigma}{\sqrt{n}}}; \mu = -z_{\alpha/2} \frac{\sigma}{\sqrt{n}} + M$$

Confidence Interval of Effect Size

- Example → One sample z-test

$$CI_{1-\alpha} \text{ of } \mu_1 = M \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\delta = \frac{\mu_1 - \mu_0}{\sigma} \quad d = \frac{M_1 - \mu_0}{\sigma}$$

$$CI_{1-\alpha} \text{ of } \mu_1 - \mu_0 = M - \mu_0 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$CI_{1-\alpha} \text{ of } \frac{1}{\sigma} (\mu_1 - \mu_0) = \frac{1}{\sigma} \left(M - \mu_0 \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

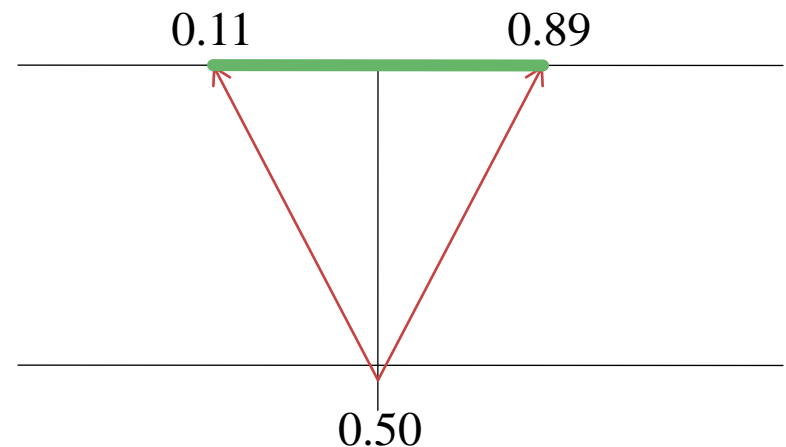
$$CI_{1-\alpha} \text{ of } \frac{\mu_1 - \mu_0}{\sigma} = \frac{M - \mu_0}{\sigma} \pm \frac{1}{\cancel{\sigma}} \left(z_{\alpha/2} \frac{\cancel{\sigma}}{\sqrt{n}} \right)$$

$$CI_{1-\alpha} \text{ of } \delta = d \pm \frac{z_{\alpha/2}}{\sqrt{n}}$$

Confidence Interval of Effect Size

- $n = 25$ students taking SAT prep course
- $M = 550$
- $\sim N(500, 100)$

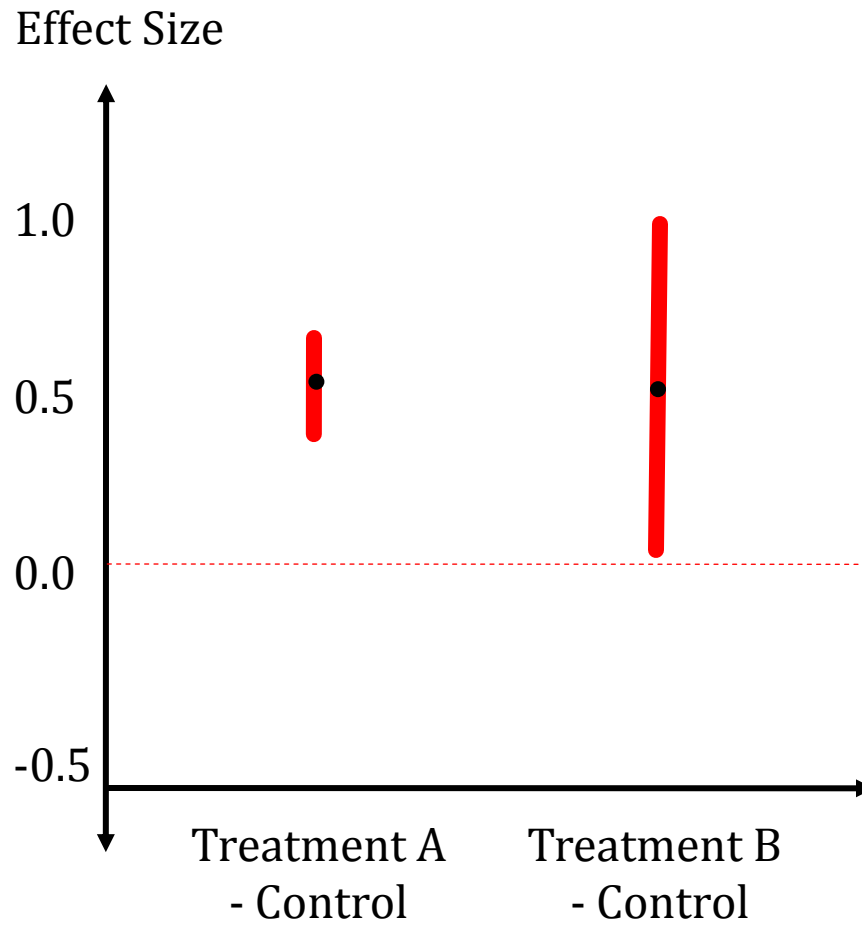
$$\begin{aligned} CI_{1-\alpha} \text{ of } \delta &= d \pm \frac{z_{\alpha/2}}{\sqrt{n}} \\ &= \frac{550 - 500}{100} \pm \frac{1.96}{\sqrt{25}} \\ &= 0.11 \text{ to } 0.89 \end{aligned}$$



Importance

- Precision

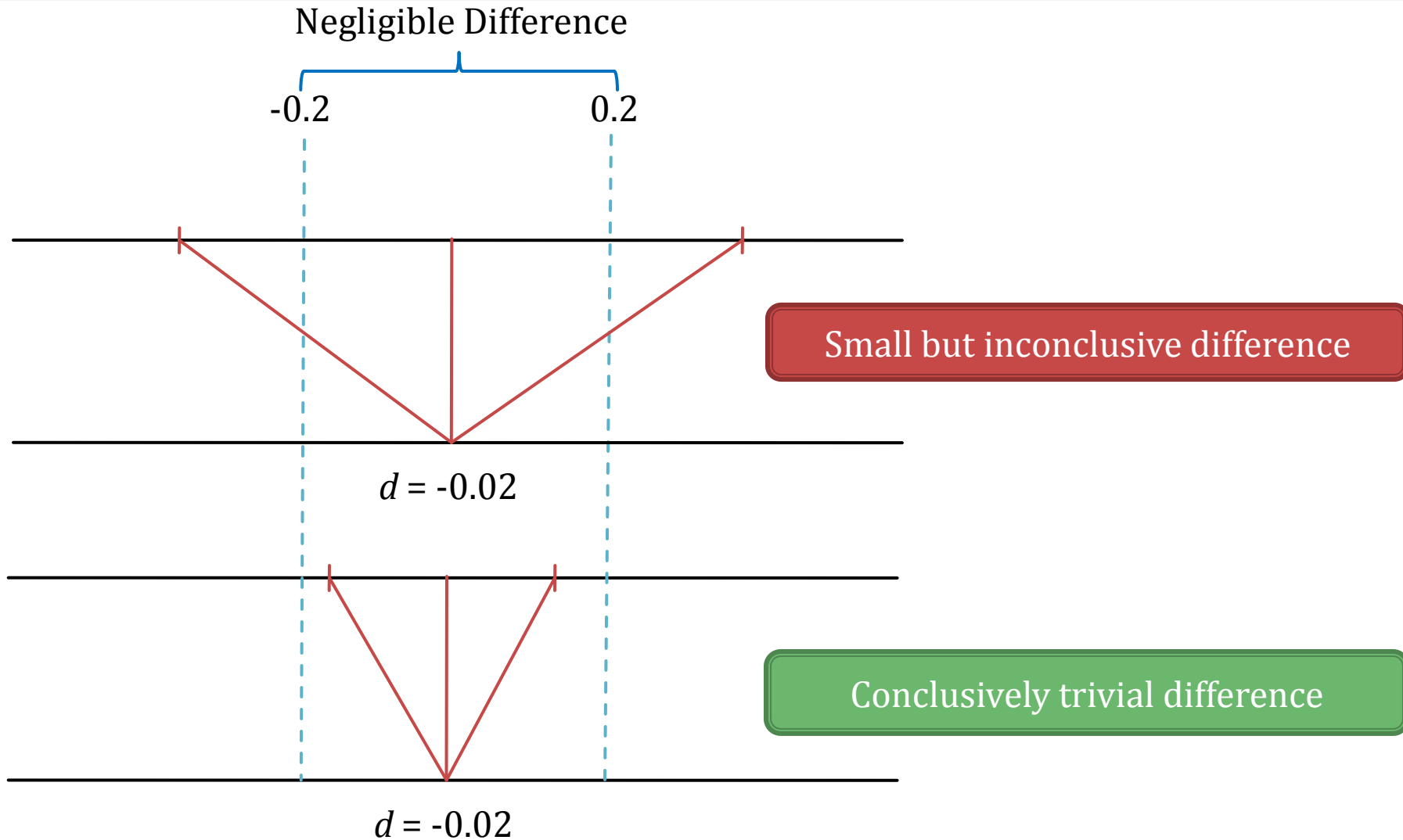
Precision



Importance

- Precision
- Accept null hypothesis?

Accepting the Null Hypothesis

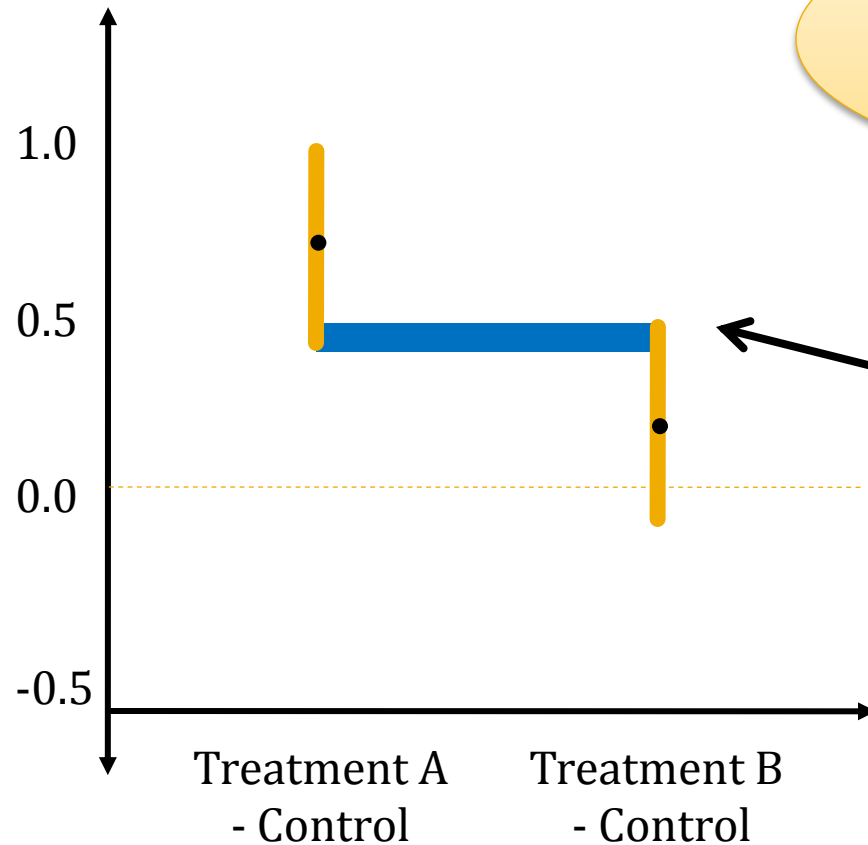


Importance

- Precision
- Accept null hypothesis?
- Compare study results

Compare Study Results

Effect Size



See Cumming and Finch
(2005) rule for two
independent samples

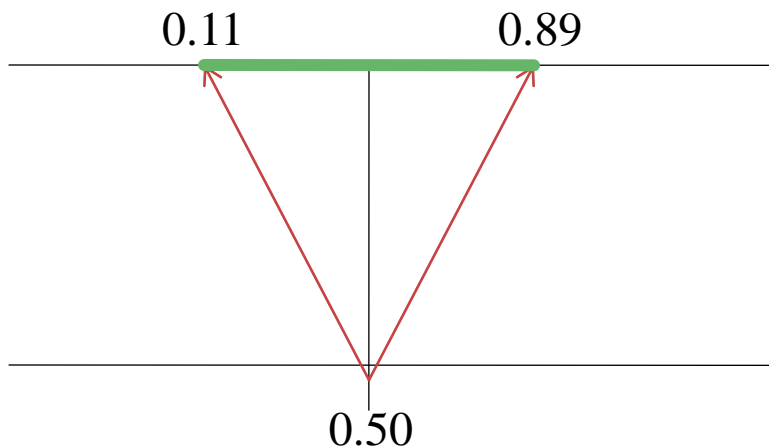
Overlap less than
half of margin error

$p < .05$

Importance

- Precision
- Accept null hypothesis?
- Compare study results
- Test a specific magnitude of effect

Confidence Interval of Effect Size



$$H_0: \delta = -0.2$$

$$H_0: \delta = 0.0$$

$$H_0: \delta = 0.2$$

$$H_0: \delta = 0.4$$

$$H_0: \delta = 0.6$$

$$H_0: \delta = 0.8$$

$$H_0: \delta = 1.0$$

$$H_0: \delta = 1.2$$

Significant, $p < .05$

Not significant, $p > .05$

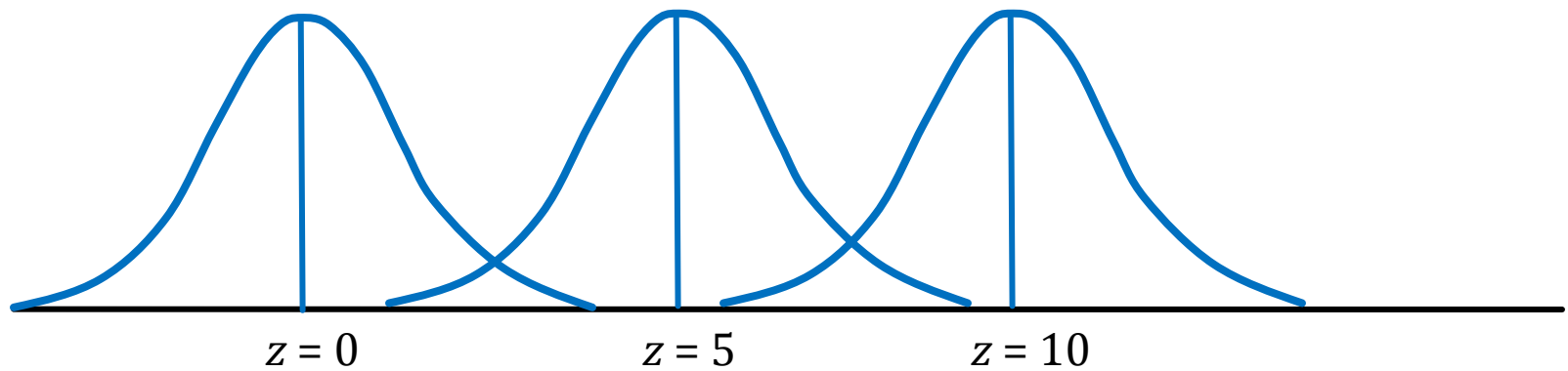
Significant, $p < .05$

Confidence Interval of Effect Size

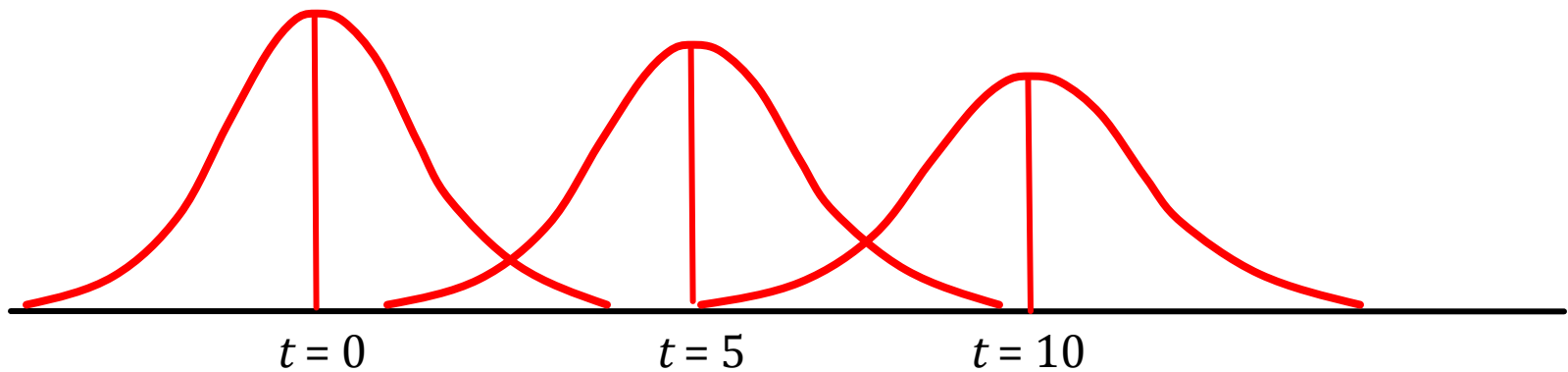
- Problem of Confidence Interval of Effect Size
 - Beyond z-distribution, using noncentral distribution

Beyond the z-Distribution

z-distribution

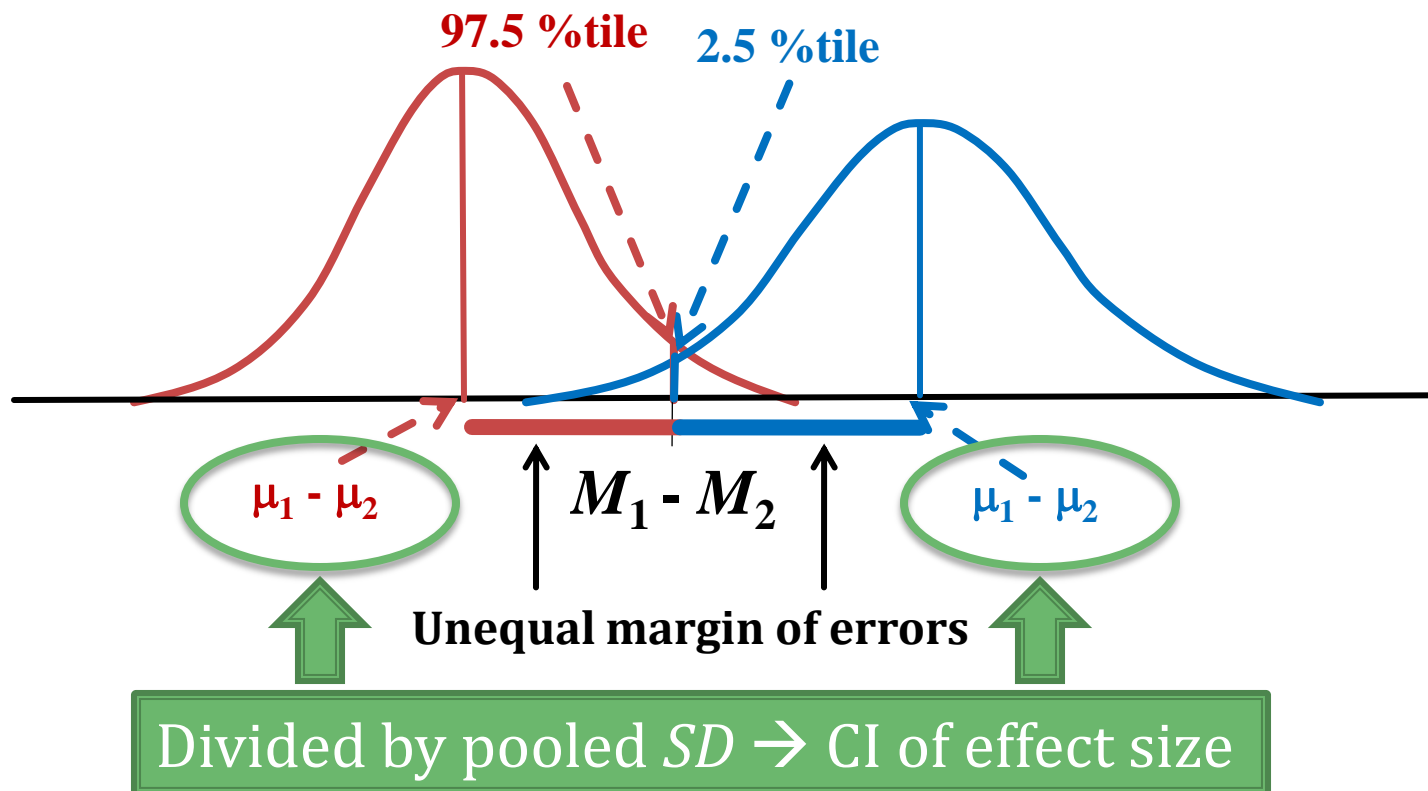


t -distribution ($df = 40$)



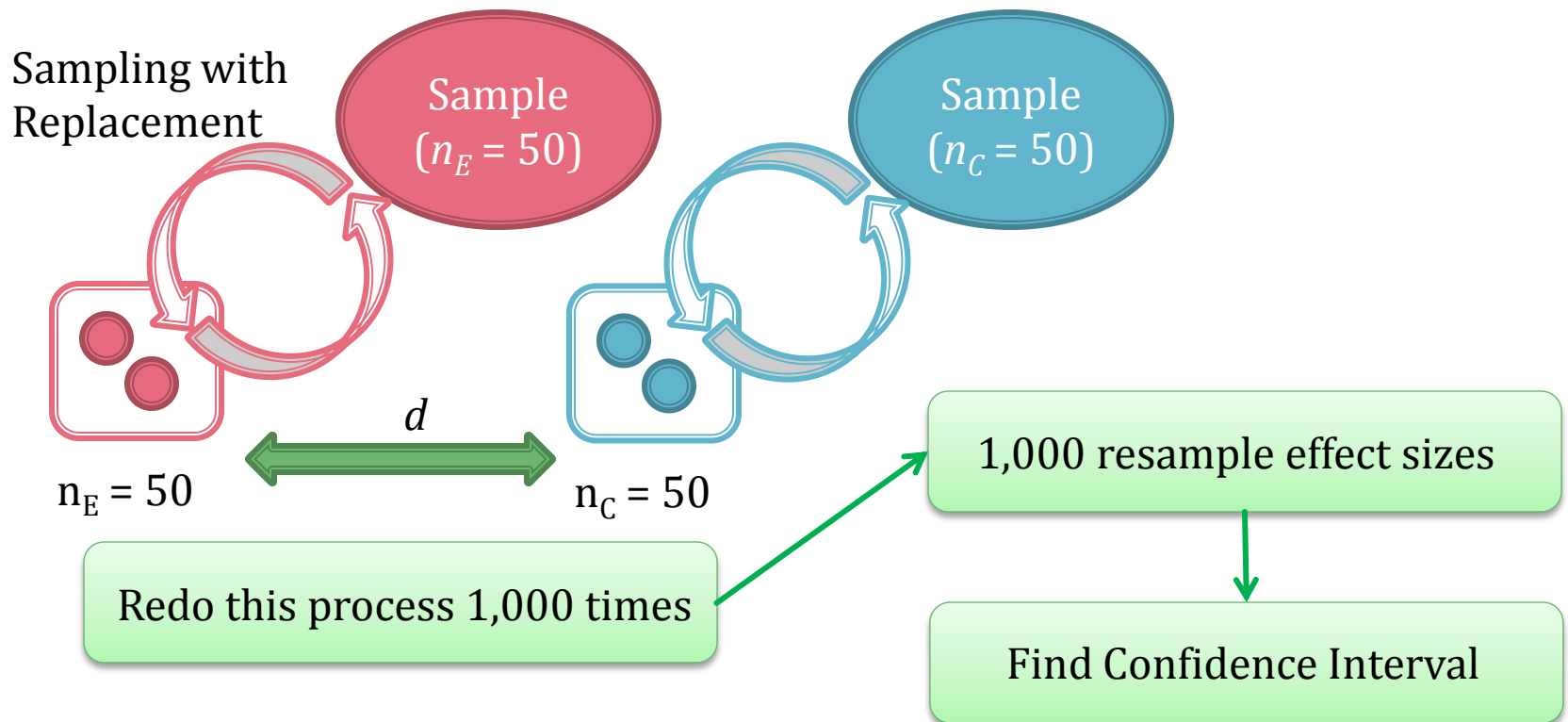
Non-symmetrical Noncentral distributions

- Example → Independent t -test
 - CI of a difference between independent means



Problem

- Difficulty of analytic solution
- Bootstrapping (Resampling)



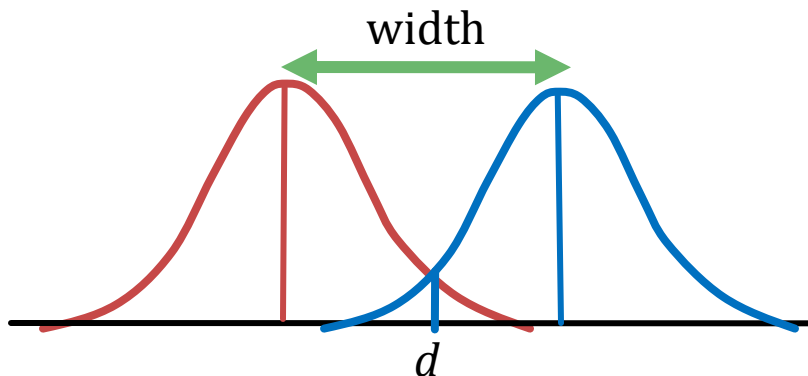
Accuracy in Effect Size Estimation

- Plan for a specific width
- Analytic approach
 - One-sample z-test

$$CI_{1-\alpha} \text{ of } \delta = d \pm \frac{z_{\alpha/2}}{\sqrt{n}}$$



$$\text{Margin of error } (e_\delta) = \frac{z_{\alpha/2}}{\sqrt{n}}$$
$$\text{Width } (w) = 2e_\delta$$

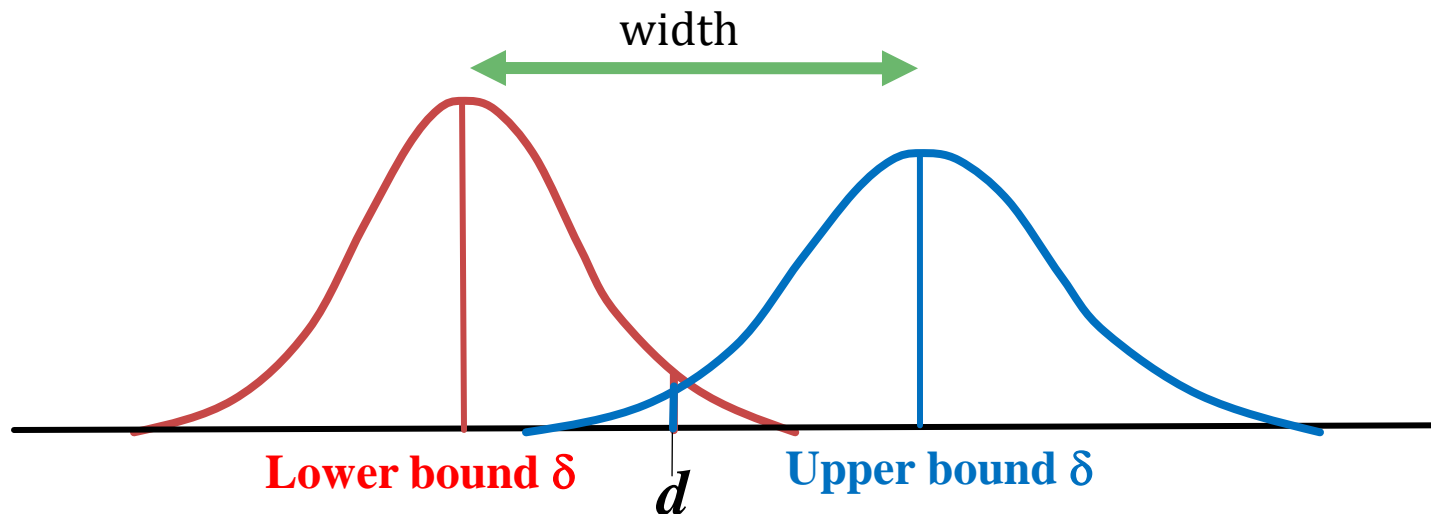


$$n = \left(\frac{z_{\alpha/2}}{e_\delta} \right)^2$$

$$n = \left(\frac{2z_{\alpha/2}}{w} \right)^2$$

Accuracy in Effect Size Estimation

- Independent t -test
- Kelley and Rausch (2006)
 - Used computer iteration
 - Assume equal sample size between groups



Accuracy in Effect Size Estimation

Find expected δ and desired width

Find initial n

Add n by 1 for each group and compute CI of effect size

Stop when width $<$ desired width

Accuracy in Effect Size Estimation

- Table of appropriate sample of *each group* for 95% CI of effect size

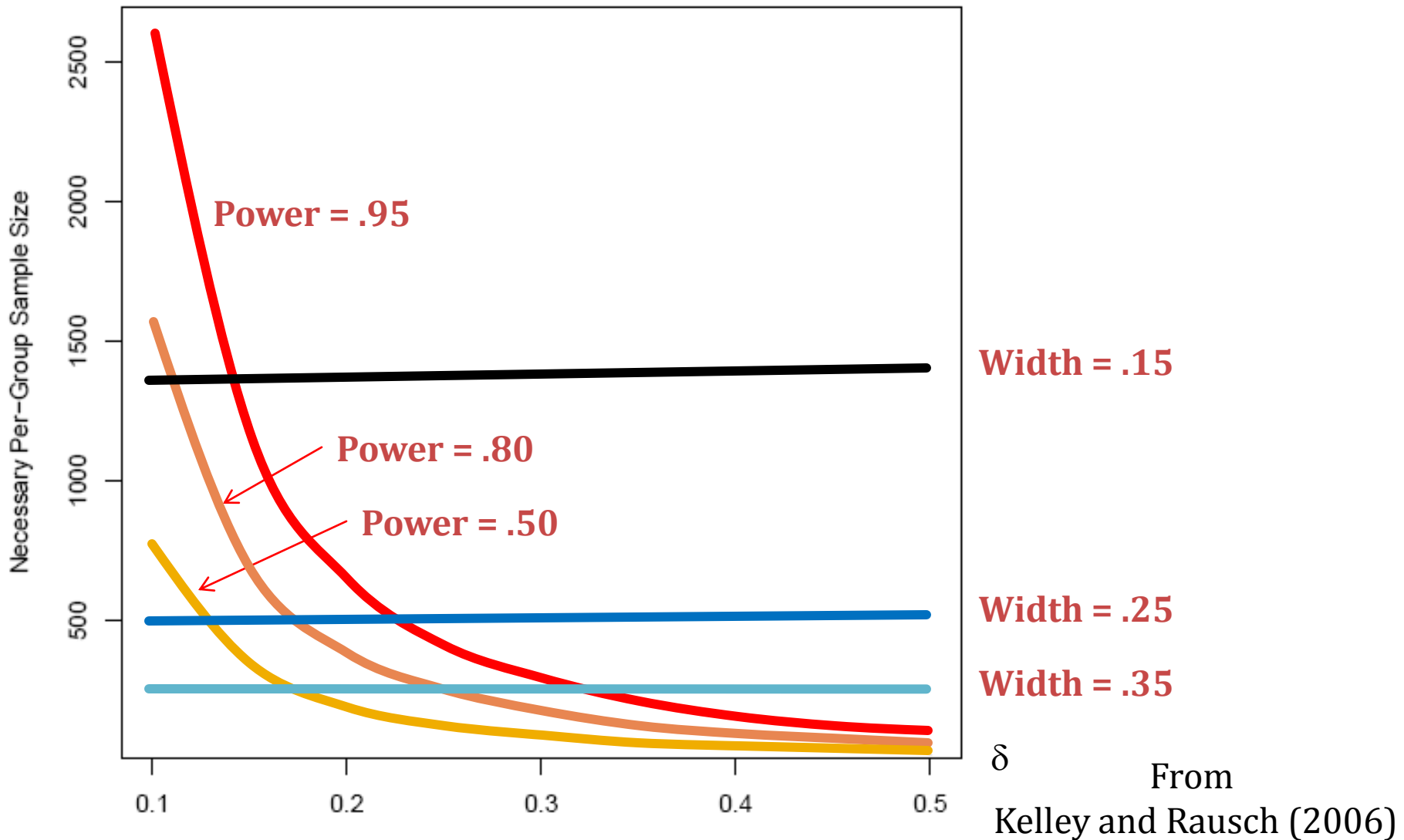
<i>width \ effect size</i>	0.10	0.20	0.50	0.80	1.00
0.10	3078	3089	3170	3320	3458
0.20	770	773	793	830	865
0.30	342	346	353	369	385
0.40	193	194	199	208	217
0.50	124	124	127	133	139
0.60	86	86	89	93	97
0.70	63	64	65	68	71

From
Kelley and Rausch (2006)

Accuracy in Effect Size Estimation

- Relationship between
 - Accuracy in effect size estimation
 - Power analytic technique

Accuracy in Effect Size Estimation



Critics

- Small width $\rightarrow n$ is huge
- Independence of error terms
- Not flexible (such as group intervention)
- Multilevel designs
 - Three strategies

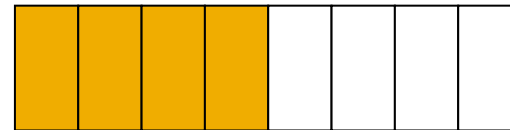
Alternative Designs

Treatment Control



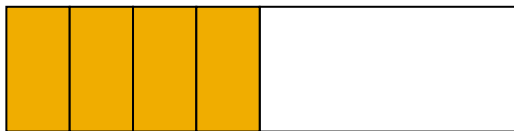
Single-site experiment

Treatment Control



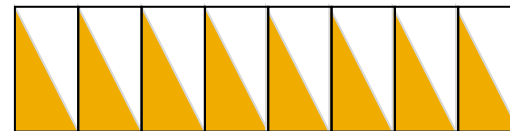
Cluster-randomized design

Treatment Control



Cluster-randomized design in which control participants are ungrouped

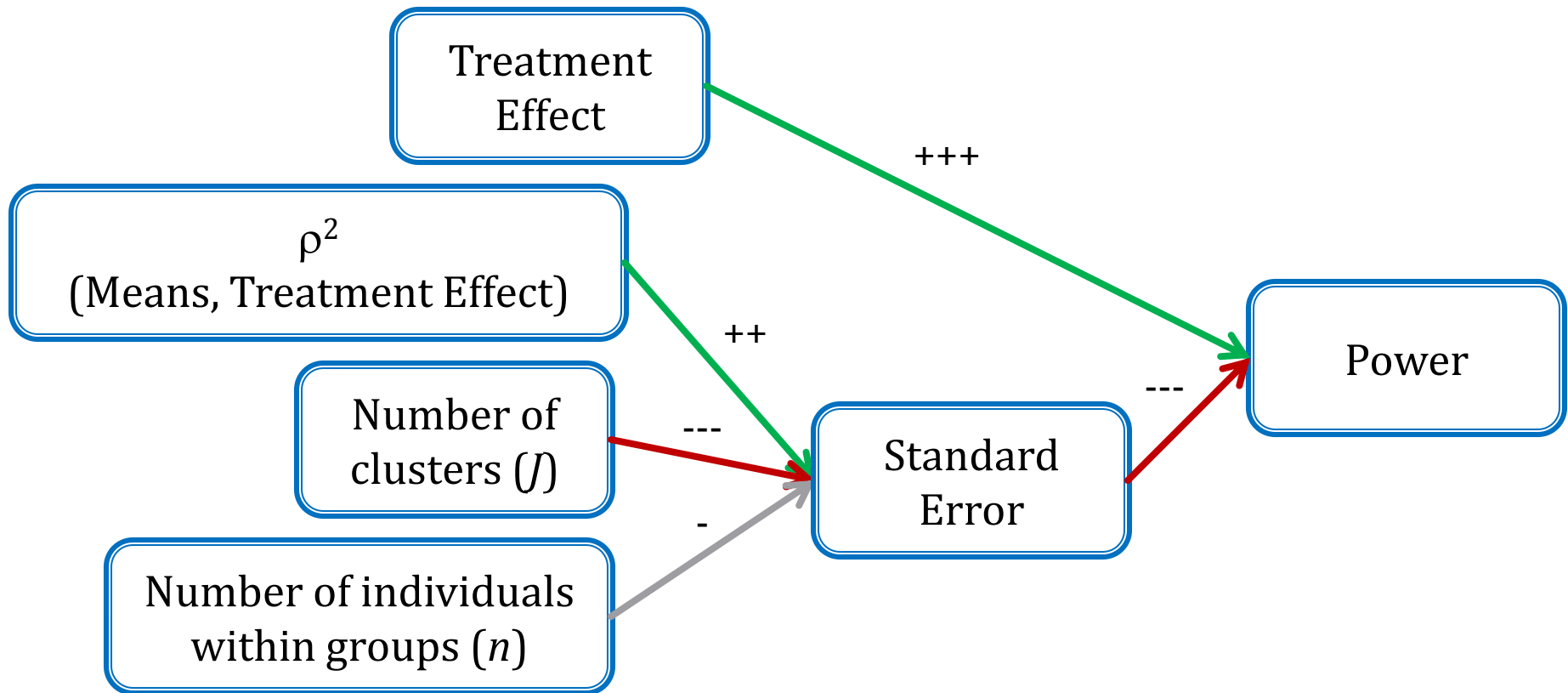
Mixed between treatment and control



Treatment Control

Multisite Randomized Trials

Previous Studies about Power



Single-site
Experiment

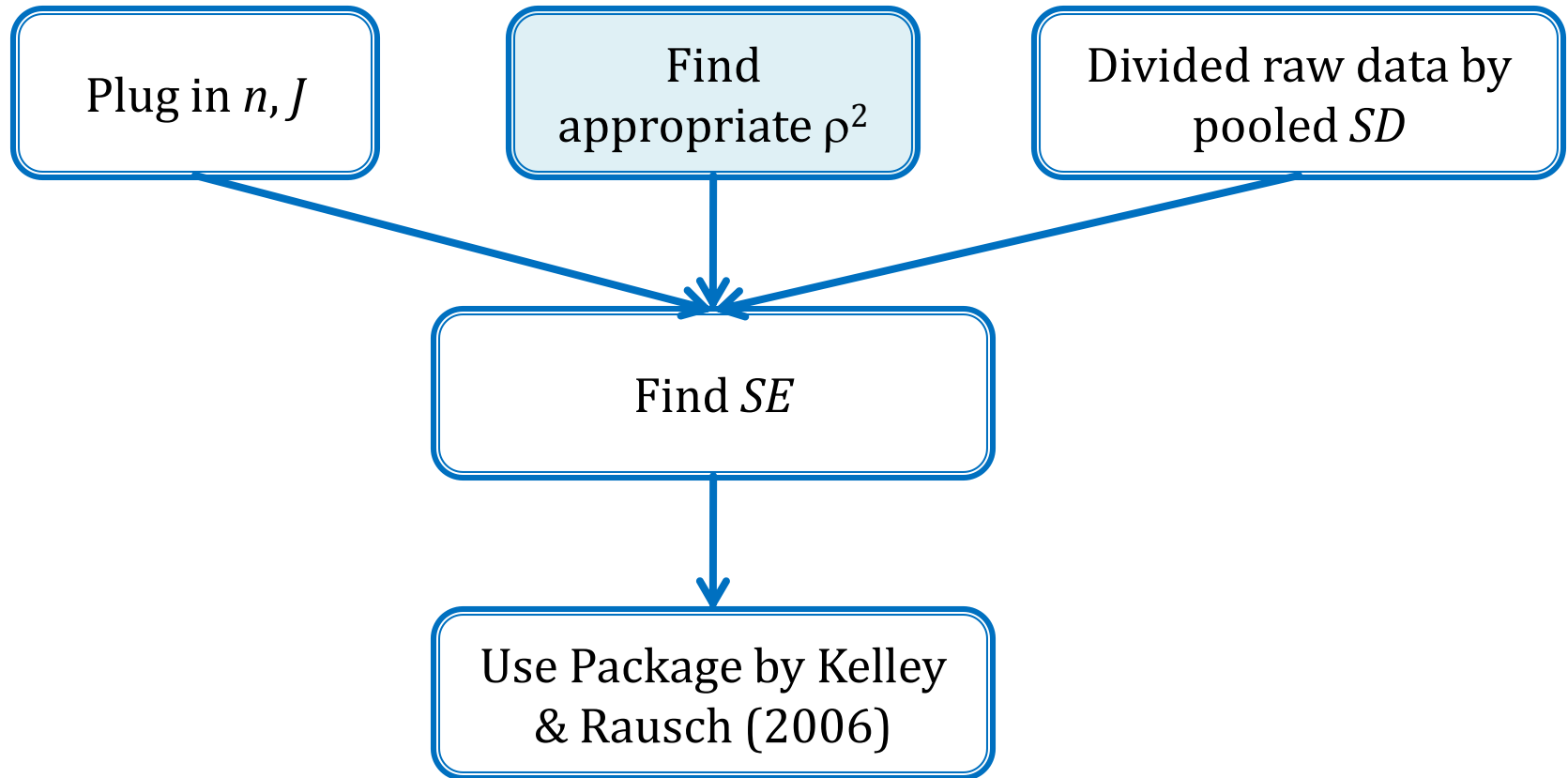
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Multisite
Experiment

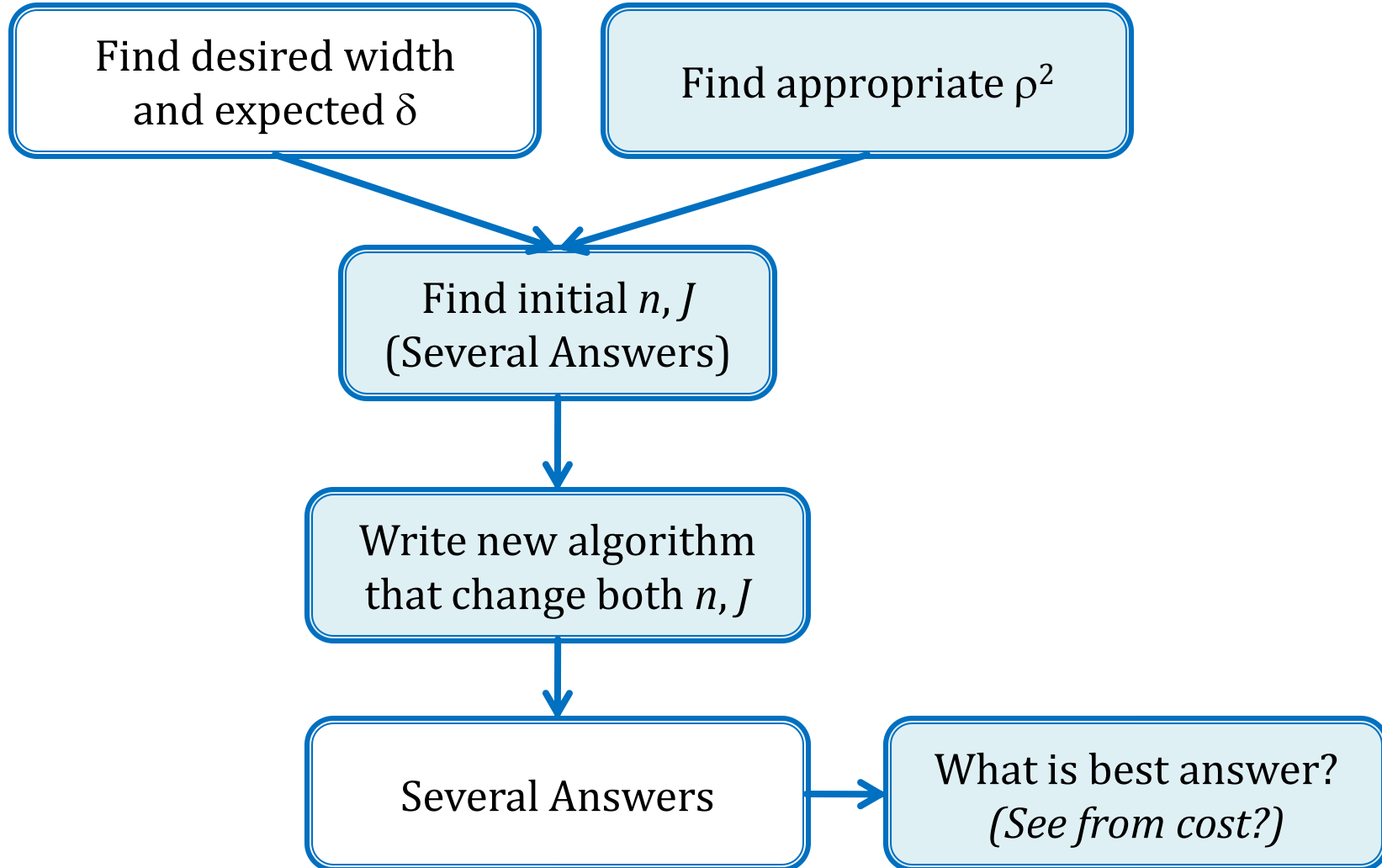
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Cluster
randomized design

Find CI of Effect Size



Find Sample Sizes for desired width



Current Problems

- Different Intraclass correlation
 - Group means
 - Treatment effects
- Incongruent formulas in *SE* of multisite experiments
- Build algorithm for R package

Alternative Thesis Topics

- Group-based intervention when control participants are ungrouped
- Costs
 - New cluster or new individuals
 - Experiment or control participants
- Control-group contamination effect

Questions, Answers, and Comments
