One-Sample Chi-square Test for the Variance

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Hypothesis Testing for Sample Variance

Because of sampling error, sample variance will not be equal to population variance.

Although the population correlation coefficient is equal to zero, the correlation statistics is not equal to zero by chance.

What is the value that sample variance is unlikely to be randomly drawn from specified population?

Null hypothesis $H_0: \sigma^2 = \sigma_0^2$

Alternative hypothesis $H_1: \sigma^2 \neq \sigma_0^2$ (Two-tailed)

$H_1: \sigma^2 > \sigma_0^2;\ \sigma^2 < \sigma_0^2$ (One-tailed)

If null hypothesis is true,

$$\frac{\hat{\sigma}^2}{\sigma_0^2} = 1$$

However, when random sampling from the population, the sample variance is not equal to population variance and then this ratio is not equal to 1.

$$\chi^2 = (n-1)\frac{\hat{\sigma}^2}{\sigma_0^2}$$

If null hypothesis is true and the random samples are drawn, the ratios will distributed like Chi-square distribution with *n*-1 degree of freedom.

Author Note
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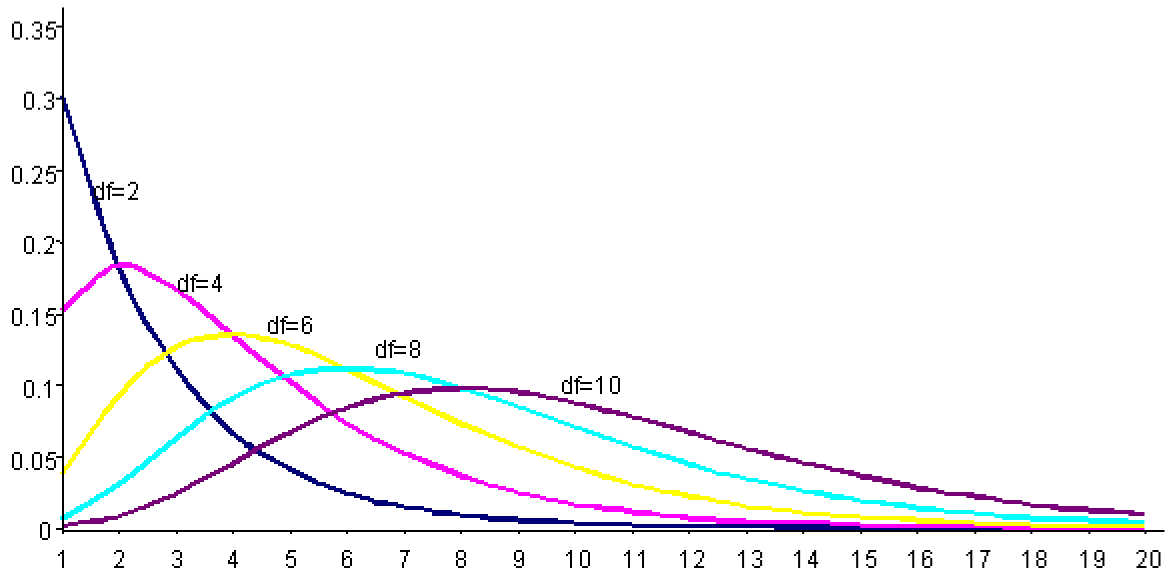
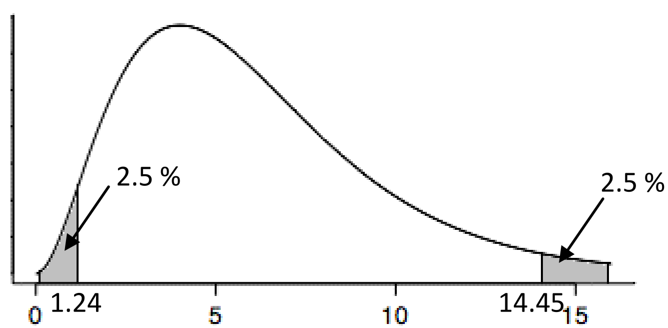


$$E(\chi^2) = df = n - 1$$

The χ^2 distribution, like the *t* distribution, is actually a family of distributions whose shape depends on its degrees of freedom.

The χ^2 distribution is positively skewed. If *df* increases, the graph will approach normal distribution.

How much extreme of this ratio will be defined as unlikely, if the null hypothesis is true.



The critical value and critical region can be determined by Table D4 in Kirk (2008) or MS Excel function

CHIINV(probability, *df*)

For example, CHIINV(.975, 6) = 1.24; CHIINV(.025, 6) = 14.45.

Another MS Excel function used for finding *p* value is

One-tailed *p* value CHIDIST(*x*, *df*)

Two-tailed p value $2 \times$ CHIDIST(x, df)

For example, One-tailed p value is CHIDIST(14.45,6) = 0.025

The confidence interval based on this null hypothesis testing is

A two-sided 100(1-α) % confidence interval for σ_1^2/σ_2^2 is given by

$$\frac{(n-1)\hat{\sigma}^2}{\chi^2_{\alpha/2\ (Right),df}} < \sigma^2 < \frac{(n-1)\hat{\sigma}^2}{\chi^2_{\alpha/2\ (Left),df}}$$

Lower and upper one-sided 100(1-α) % confidence intervals for σ_1^2/σ_2^2 are given by

$$\frac{(n-1)\hat{\sigma}^2}{\chi^2_{\alpha\ (Right),df}} < \sigma^2 \qquad \text{and} \qquad \sigma^2 < \frac{(n-1)\hat{\sigma}^2}{\chi^2_{\alpha(Left),df}}$$

Example

Assumption of this formula

1) A random sample of n observations is obtained from the population of interest.
2) The population is normally distributed

The one-sample χ^2 test for the variance is not robust to violation of normality assumption.