ANOVA for Factorial Design

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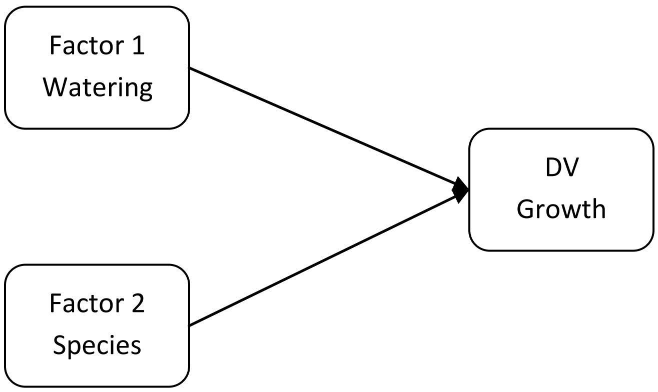
Sometimes, the researchers want to test hypotheses about two or more independent variables simultaneously in a single experiment.

In this lecture, the two-way factorial design (two independent variables) will be discussed.

For example,



Factor 1: A little

Much

Factor 2: Devil's Ilvy (Plu-dang)

Cactur (Kra-bong-petch)

DV: Growth (cm)

Factor 2

		Group 1		Group 2		Group 3		Average
Factor 1	**Group 1**	X_{111} X_{211} X_{311} X_{411} X_{511}	$\bar{X}_{.11}$	X_{112} X_{212} X_{312} X_{412} X_{512}	$\bar{X}_{.12}$	X_{113} X_{213} X_{313} X_{413} X_{513}	$\bar{X}_{.13}$	$\bar{X}_{.1.}$
	Group2	X_{121} X_{221} X_{321} X_{421} X_{521}	$\bar{X}_{.21}$	X_{122} X_{222} X_{322} X_{422} X_{522}	$\bar{X}_{.22}$	X_{123} X_{223} X_{323} X_{423} X_{523}	$\bar{X}_{.23}$	$\bar{X}_{.2.}$
	Average	$\bar{X}_{..1}$		$\bar{X}_{..2}$		$\bar{X}_{..3}$		$\bar{X}_{...}$

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ANOVA as a Regression Analysis

No Predictor

In this analysis, there are two independent variables: motivator factor (1= low, 2 = high) and hygiene factor (1= low, 2 = high). The dependent variable is job performance.

The prediction score is arithmetic mean.

Arithmetic Mean

Score	Performance	Motivator	Hygiene	Prediction Score	Error of Prediction	Squared error
1	65	1	1	71	-6	36
2	55	1	1	71	-16	256
3	70	1	1	71	-1	1
4	60	1	1	71	-11	121
5	65	1	1	71	-6	36
6	70	1	2	71	-1	1
7	65	1	2	71	-6	36
8	60	1	2	71	-11	121
9	75	1	2	71	4	16
10	65	1	2	71	-6	36
11	60	2	1	71	-11	121
12	70	2	1	71	-1	1
13	75	2	1	71	4	16
14	65	2	1	71	-6	36
15	75	2	1	71	4	16
16	80	2	2	71	9	81
17	95	2	2	71	24	576
18	75	2	2	71	4	16
19	85	2	2	71	14	196
20	90	2	2	71	19	361
Total					**0**	**2080**

$SS_{error} = SS_{Total} = 2080$

$$SS_{\text{total}} = 2080$$

One Grouping Variable: Factor 1

If there is one categorical variable as an independent variable, the values that can predict all value leaving least error are group means.

$$X' = \bar{X}_{.j.} = \sum_{k=1}^{t} \frac{1}{t} \sum_{i=1}^{n_{jk}} \frac{1}{n_{jk}} X_{ijk}$$

The group means is equal to the sum of grand mean and treatment effect.

$$X' = \bar{X}_{...} + \left(\bar{X}_{.j.} - \bar{X}_{...}\right)$$

F_1 Group Mean

Factor 1

Score	Performance	Motivator	Hygiene	Factor 1 Effect	Prediction Score	Error of Prediction	Squared error
1	65	1	1	-6	65	0	0
2	55	1	1	-6	65	-10	100
3	70	1	1	-6	65	5	25
4	60	1	1	-6	65	-5	25
5	65	1	1	-6	65	0	0
6	70	1	2	-6	65	5	25
7	65	1	2	-6	65	0	0
8	60	1	2	-6	65	-5	25
9	75	1	2	-6	65	10	100
10	65	1	2	-6	65	0	0
11	60	2	1	6	77	-17	289
12	70	2	1	6	77	-7	49
13	75	2	1	6	77	-2	4
14	65	2	1	6	77	-12	144
15	75	2	1	6	77	-2	4
16	80	2	2	6	77	3	9
17	95	2	2	6	77	18	324
18	75	2	2	6	77	-2	4
19	85	2	2	6	77	8	64
20	90	2	2	6	77	13	169
Total						**0**	**1360**

$SS_{error} = 1360$

$$SS_{\text{group}} = 2080 - 1360 = 720$$

$$\eta^2 = 1 - \frac{1360}{2080} = 0.346$$

One Grouping Variable: Factor 2

If there is one categorical variable as an independent variable, the values that can predict all value leaving least error are group means.

$$X' = \bar{X}_{..k} = \sum_{j=1}^{s} \frac{1}{s} \sum_{i=1}^{n_{jk}} \frac{1}{n_{jk}} X_{ijk}$$

The group means is equal to the sum of grand mean and treatment effect.

$$X' = \bar{X}_{...} + (\bar{X}_{..k} - \bar{X}_{...})$$

Factor 2

F_2 Group Mean

Score	Performance	Motivator	Hygiene	Factor 2 Effect	Prediction Score	Error of Prediction	Squared error
1	65	1	1	-5	66	-1	1
2	55	1	1	-5	66	-11	121
3	70	1	1	-5	66	4	16
4	60	1	1	-5	66	-6	36
5	65	1	1	-5	66	-1	1
6	70	1	2	5	76	-6	36
7	65	1	2	5	76	-11	121
8	60	1	2	5	76	-16	256
9	75	1	2	5	76	-1	1
10	65	1	2	5	76	-11	121
11	60	2	1	-5	66	-6	36
12	70	2	1	-5	66	4	16
13	75	2	1	-5	66	9	81
14	65	2	1	-5	66	-1	1
15	75	2	1	-5	66	9	81
16	80	2	2	5	76	4	16
17	95	2	2	5	76	19	361
18	75	2	2	5	76	-1	1
19	85	2	2	5	76	9	81
20	90	2	2	5	76	14	196
Total						**0**	**1580**

SS_{error} = 1580

$$SS_{group} = 2080 - 1580 = 500$$

$$\eta^2 = 1 - \frac{1580}{2080} = 0.24$$

Two Grouping Variable

If there is two categorical variables as an independent variable, the values that can predict all value leaving least error are cell means.

$$X' = \bar{X}_{.jk} = \sum_{i=1}^{n_{jk}} \frac{1}{n_{jk}} X_{ijk}$$

The group means is equal to the sum of grand mean and cell effect.

$$X' = \bar{X}_{...} + \left(\bar{X}_{.jk} - \bar{X}_{...}\right)$$

Cell Mean

Score	Performance	Motivator	Hygiene	Prediction Score	Error of Prediction	Squared error
1	65	1	1	63	2	4
2	55	1	1	63	-8	64
3	70	1	1	63	7	49
4	60	1	1	63	-3	9
5	65	1	1	63	2	4
6	70	1	2	67	3	9
7	65	1	2	67	-2	4
8	60	1	2	67	-7	49
9	75	1	2	67	8	64
10	65	1	2	67	-2	4
11	60	2	1	69	-9	81
12	70	2	1	69	1	1
13	75	2	1	69	6	36
14	65	2	1	69	-4	16
15	75	2	1	69	6	36
16	80	2	2	85	-5	25
17	95	2	2	85	10	100
18	75	2	2	85	-10	100
19	85	2	2	85	0	0
20	90	2	2	85	5	25
Total					**0**	**680**

$SS_{error} = 680$

$$SS_{\text{group}} = 2080 - 680 = 1400$$

$$\eta^2 = 1 - \frac{680}{2080} = 0.673$$

If replaced the cell means for prediction to the sum of factor 1 and factor 2 effects

$$X' = \bar{X}_{...} + \left(\bar{X}_{.j.} - \bar{X}_{...}\right) + (\bar{X}_{..k} - \bar{X}_{...})$$

Cell Mean

Score	Performance	Motivator	Hygiene	Factor 1 Effect	Factor 2 Effect	Prediction Score	Error of Prediction	Squared error
1	65	1	1	-6	-5	60	5	25
2	55	1	1	-6	-5	60	-5	25
3	70	1	1	-6	-5	60	10	100
4	60	1	1	-6	-5	60	0	0
5	65	1	1	-6	-5	60	5	25
6	70	1	2	-6	5	70	0	0
7	65	1	2	-6	5	70	-5	25
8	60	1	2	-6	5	70	-10	100
9	75	1	2	-6	5	70	5	25
10	65	1	2	-6	5	70	-5	25
11	60	2	1	6	-5	72	-12	144
12	70	2	1	6	-5	72	-2	4
13	75	2	1	6	-5	72	3	9
14	65	2	1	6	-5	72	-7	49
15	75	2	1	6	-5	72	3	9
16	80	2	2	6	5	82	-2	4
17	95	2	2	6	5	82	13	169
18	75	2	2	6	5	82	-7	49
19	85	2	2	6	5	82	3	9
20	90	2	2	6	5	82	8	64
Total							0	860

$SS_{error} = 860$

$$SS_{\text{factor 1+2}} = 2080 - 860 = 1220 = SS_1 + SS_2$$

$$\eta^2 = 1 - \frac{680}{2080} = 0.587 = \eta_1^2 + \eta_2^2$$

You will see that

$$SS_{\text{cells}} \neq SS_{\text{factor 1+2}}$$

$$\bar{X}_{.jk} \neq \bar{X}_{...} + \left(\bar{X}_{.j.} - \bar{X}_{...}\right) + \left(\bar{X}_{..k} - \bar{X}_{...}\right)$$

$$\bar{X}_{.jk} = \bar{X}_{...} + \left(\bar{X}_{.j.} - \bar{X}_{...}\right) + \left(\bar{X}_{..k} - \bar{X}_{...}\right) + \left(\bar{X}_{.jk} - \bar{X}_{.j.} - \bar{X}_{..k} + \bar{X}_{...}\right)$$

If $SS_{\text{cells}} \neq SS_{\text{factor 1+2}}$,

$$\bar{X}_{.jk} - \bar{X}_{.j.} - \bar{X}_{..k} + \bar{X}_{...} \neq 0$$

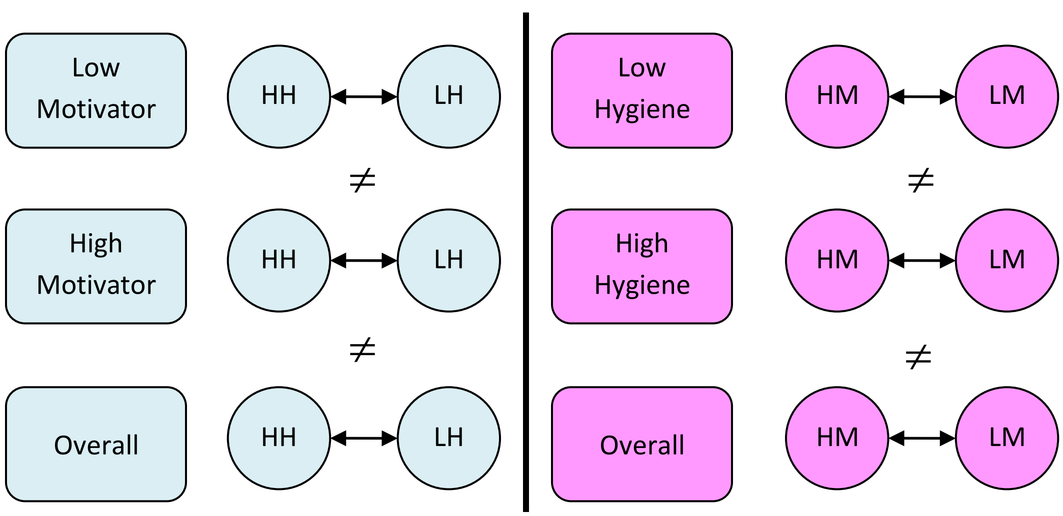
$$\left(\bar{X}_{.jk} - \bar{X}_{.j.}\right) - \left(\bar{X}_{..k} - \bar{X}_{...}\right) \neq 0 \qquad and \qquad \left(\bar{X}_{.jk} - \bar{X}_{..k}\right) - \left(\bar{X}_{.j.} - \bar{X}_{...}\right) \neq 0$$

$$\left(\bar{X}_{.jk} - \bar{X}_{.j.}\right) \neq \left(\bar{X}_{..k} - \bar{X}_{...}\right) \qquad and \qquad \left(\bar{X}_{.jk} - \bar{X}_{..k}\right) \neq \left(\bar{X}_{.j.} - \bar{X}_{...}\right)$$

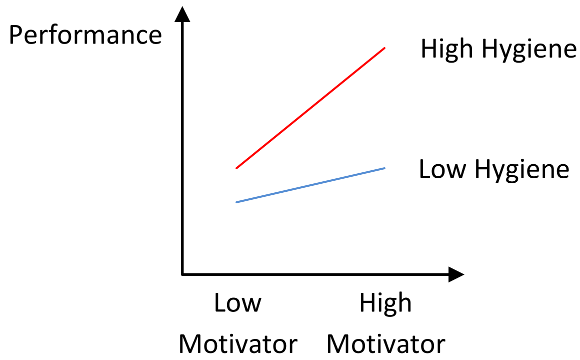
The factor 2 in treatment *j* is not equal the overall factor 2 effect.

The factor 1 in treatment *k* is not equal the overall factor 1 effect.

For example,



It is a moderator or interaction effect; that is, the effect of A is not equal in each B group and the effect of B is not equal in each A group.



The lost sum of squared deviation is

$$SS_{\text{cells}} - SS_{\text{factor } 1+2} = SS_{1\times 2}$$

$$SS_{\text{cells}} = SS_1 + SS_2 + SS_{1\times 2}$$

$$SS_{\text{total}} = SS_{\text{cells}} + SS_{\text{error}}$$

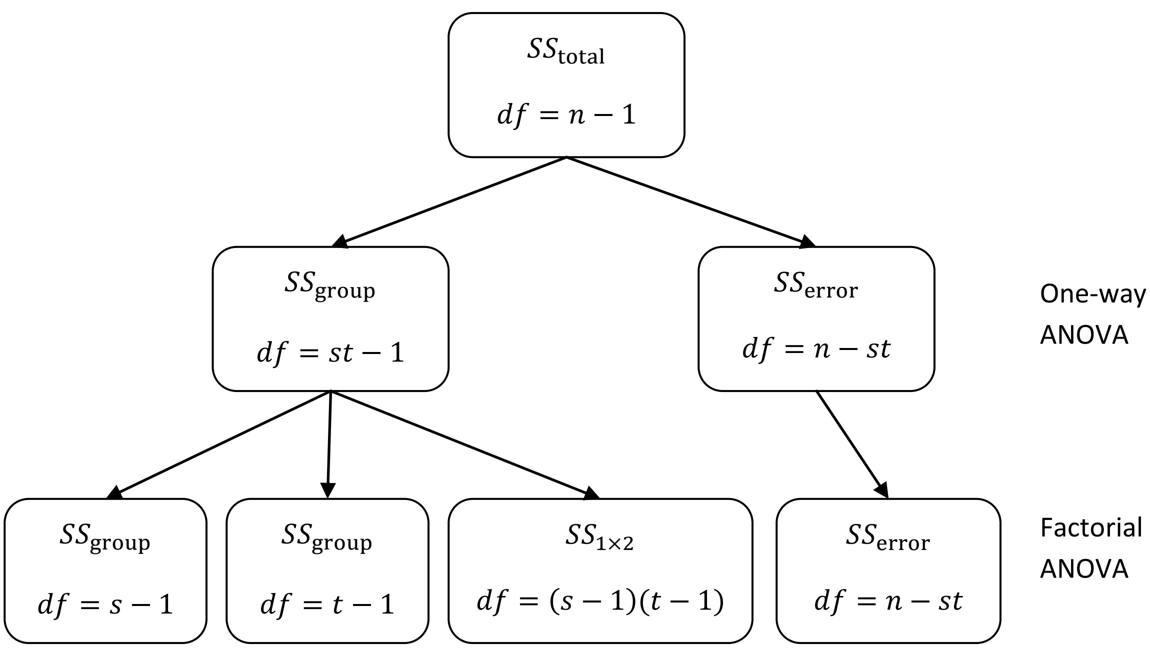
$$SS_{\text{total}} = SS_1 + SS_2 + SS_{1\times 2} + SS_{\text{error}}$$

Therefore, the sample model equation is

$$X_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \varepsilon_{i(jk)}$$

$$X_{ijk} = \bar{X}_{...} + \left(\bar{X}_{.j.} - \bar{X}_{...}\right) + \left(\bar{X}_{..k} - \bar{X}_{...}\right) + \left(\bar{X}_{.jk} - \bar{X}_{.j.} - \bar{X}_{..k} + \bar{X}_{...}\right) + \left(X_{ijk} - \bar{X}_{.jk}\right)$$

Score = Grand mean +Main effect from Factor 1+ Main effect from Factor 2 +Interaction Effect + Error effect



For example

Case 1 $65 = 71 + (-6) + (-5) + (3)$

Case 6 $70 = 71 + (-6) + (5) + (-3)$

Summary Factorial ANOVA

Score	Performance	Motivator	Hygiene	Factor 1 Effect	Factor 2 Effect	Factor 1 x 2 Effect	Prediction Score	Error of Prediction	Squared error
1	65	1	1	-6	-5	3	63	2	4
2	55	1	1	-6	-5	3	63	-8	64
3	70	1	1	-6	-5	3	63	7	49
4	60	1	1	-6	-5	3	63	-3	9
5	65	1	1	-6	-5	3	63	2	4
6	70	1	2	-6	5	-3	67	3	9
7	65	1	2	-6	5	-3	67	-2	4
8	60	1	2	-6	5	-3	67	-7	49
9	75	1	2	-6	5	-3	67	8	64
10	65	1	2	-6	5	-3	67	-2	4
11	60	2	1	6	-5	-3	69	-9	81
12	70	2	1	6	-5	-3	69	1	1
13	75	2	1	6	-5	-3	69	6	36
14	65	2	1	6	-5	-3	69	-4	16
15	75	2	1	6	-5	-3	69	6	36
16	80	2	2	6	5	3	85	-5	25
17	95	2	2	6	5	3	85	10	100
18	75	2	2	6	5	3	85	-10	100
19	85	2	2	6	5	3	85	0	0
20	90	2	2	6	5	3	85	5	25
Total								0	680

$$SS_{\text{total}} = 2080;\ SS_1 = 720;\ SS_2 = 500;\ SS_{1\times 2} = 180;\ SS_{\text{error}} = 680$$

$$\eta_1^2 = 0.346;\ \eta_2^2 = 0.240; \eta_{1\times 2}^2 = 0.086$$

Factorial-ANOVA for Testing Hypothesis

When researchers want to test hypotheses about more than one factor that affect dependent variable, the prefer statistic is Factorial ANOVA.

The hypotheses that can be tested in factorial ANOVA is the hypotheses about main effect and interaction effect.

Null hypothesis for factor 1 effect $H_0: \mu_{.1.} = \mu_{.2.} = \mu_{.3.} = \cdots = \mu_{.j.}$

Null hypothesis for factor 2 effect $H_0: \mu_{..1} = \mu_{..2} = \mu_{..3} = \cdots = \mu_{..j}$

Null hypothesis for interaction effect of factor 1 and 2

H_0: Effect of factor 1 is equal in each group of factor 2.

H_0: Effect of factor 2 is equal in each group of factor 1.

Alternative hypothesis for factor 1 effect $H_1: \mu_{.i.} \neq \mu_{.j.}$

Alternative hypothesis for factor 2 effect $H_1: \mu_{..i} \neq \mu_{..j}$

Alternative hypothesis for interaction effect of factor 1 and 2

H_0: Effect of factor 1 is not equal in each group of factor 2.

H_0: Effect of factor 2 is not equal in each group of factor 1.

The total degrees of freedom in ANOVA are divided in four components.

$$df_{\text{total}} = n - 1$$

$$df_1 = s - 1$$

$$df_2 = t - 1$$

$$df_{\text{error}} = n - st$$

$$df_{1\times2} = df_{\text{total}} - df_1 - df_2 - df_{\text{error}} = (s-1)(t-1)$$

$$df_{\text{total}} = df_1 + df_2 + df_{1\times2} + df_{\text{error}}$$

The means of squared error (often called mean squares) are the sum of squared error divided by its degree of freedom.

$$\hat{\sigma}^2 = MS_{\text{total}} = \frac{SS_X}{df_X}$$

$$MS_1 = \frac{SS_1}{df_1}$$

$$MS_2 = \frac{SS_2}{df_2}$$

$$MS_{1\times2} = \frac{SS_{1\times2}}{df_{1\times2}}$$

$$MS_{\text{error}} = \frac{SS_{\text{error}}}{df_{\text{error}}}$$

Testing for Main Effect Differences

	Main Effect: Factor 1	Main Effect: Factor 2	Interaction Effect: Factor 1 x 2
Null Hypothesis	$H_0: \alpha_j = 0$ for all *j*	$H_0: \beta_k = 0$ for all *k*	$H_0: (\alpha\beta)_{jk} = 0$ for all *j* and *k*
If H_0 is true,	$E(MS_1) = E(MS_{\text{error}}) = \sigma_\varepsilon^2$	$E(MS_2) = E(MS_{\text{error}}) = \sigma_\varepsilon^2$	$E(MS_{1\times2}) = E(MS_{\text{error}}) = \sigma_\varepsilon^2$
Then	$F_1 = \frac{MS_1}{MS_{\text{error}}} \approx 1$	$F_2 = \frac{MS_2}{MS_{\text{error}}} \approx 1$	$F_{1\times2} = \frac{MS_{1\times2}}{MS_{\text{error}}} \approx 1$
Distributed in	*F* with df_1, df_{error}	*F* with df_2, df_{error}	*F* with $df_{1\times2}$, df_{error}
If H_0 is not tenable,	$F_1 = \frac{MS_1}{MS_{\text{error}}} \geq 1$	$F_2 = \frac{MS_2}{MS_{\text{error}}} \geq 1$	$F_{1\times2} = \frac{MS_{1\times2}}{MS_{\text{error}}} \geq 1$

If the chance of type I error of the specified *F* (*p* value) is less than alpha level, the null hypothesis rejected.

The factorial ANOVA can divide the between group variance to three parts in order to interpret the meaning of the group variance: main effects of factors or interaction effect of factors

Example

One-way ANOVA design: comparing 4 means for each hygiene and motivator group

Effect	*SS*	*df*	*MS*	*F*	*p*
Between	1400	3	466.67	10.98	< .001
Error	680	16	42.50		
Total	2080	19			

The difference between groups is significant.

Two-way ANOVA design: comparing the effects from two factors (motivator and hygiene) and their interaction.

Effect	*SS*	*df*	*MS*	*F*	*p*
Motivator	720	1	720.00	16.94	.001
Hygiene	500	1	500.00	11.77	.003
Motivator x Hygiene	180	1	180.00	4.24	.056
Error	680	6	42.50		
Total	2080	19			

The interaction effect is not statistical significant. However, the both main effects is statistical significant.

The advantages of factorial ANOVA are

1) Test hypotheses about interactions.
2) The design makes efficient use of participants.

The disadvantages of factorial ANOVA are

1) If numerous treatments are included in an experiment, the number of participants required may be prohibitive.
2) The interpretation of the analysis is not straightforward if the test of the interaction is significant.
3) The use of factorial design commits a researcher to a relatively large experiment.

Assumption of repeated-measure ANOVA

1) The model equation $X_{ijk} = \mu + (\mu_{j.} - \mu) + (\mu_{.k} - \mu) + (\mu_{jk} - \mu_{j.} - \mu_{.k} - \mu) + (X_{ijk} - \mu_{jk})$ reflects all the sources of variation that affect X_{ijk}.
2) Participants are random samples from the respective populations or the participants have been randomly assigned to the treatment combinations.
3) The population for each of the *pq* treatment combinations is normally distributed.
4) The variances of each of the *pq* treatment combinations are equal.
5) The numbers of participants in each cell are equal.

The F test is robust with respect to violation of assumption 3.

The violation of assumption 4 can be replaced ANOVA by Welch procedure.

If the numbers of participants in each cell are not equal, the regression approach to factorial ANOVA may be used.

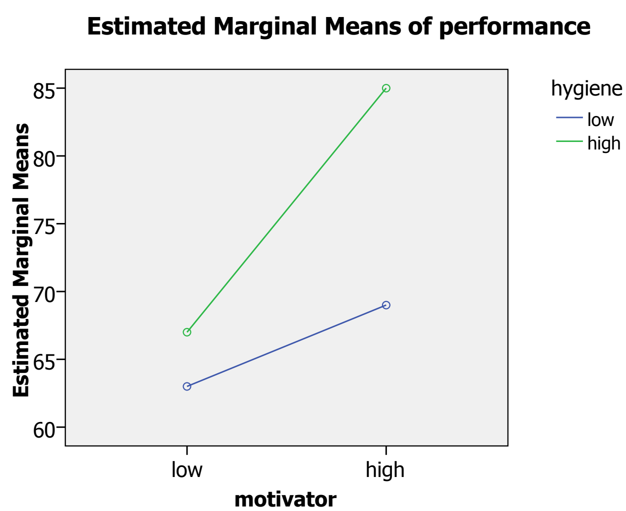
Analyzing Interaction

The nonsignificant interaction tells you that the different effect on each group is not greater than would be expected by chance.

Two treatments are said to interact if differences in performance under the levels of one treatment are different at two or more levels of the other treatment.

The presence of interaction is a signal that the interpretation of tests of the associated treatments is usually misleading and hence of little interest.

One of the useful procedures for understanding and interpreting an interaction is to graph it.



Another approach for interpreting interaction is the analysis of simple effects. This will be explained later.

Multiple Comparison Procedures

Multiple Comparison in Interaction Effects

One of the most used for interpreting interaction effect is the analysis of simple effect.

A simple effect is the effect of one factor at a given level of the other factor.

This can be conducted one-way ANOVA in specified group but used the MS_{error} in factor design instead.

In testing for simple effects we increase the number of statistical tests conducted and potentially increase the probability of a type I error.

To control the error a popular approach is to use the Bonferreni adjustment for simple effects. The Bonferreni adjustment is defined the alpha in each test equal to the preferred alpha divided by a number of contrasts.

$$\alpha_C = \frac{\alpha_{\text{overall}}}{C}$$

For example, in the analysis of hygiene and motivator factors on performance (supposed that the interaction effect is significant)

Motivator difference in each hygiene group

Difference of motivator in low hygiene ($\bar{X}_{D\text{ in low hygiene}} = 6$)

$$F(1,16) = 2.118, p = .165$$

Difference of motivator in high hygiene ($\bar{X}_{D\text{ in high hygiene}} = 18$)

$$F(1,16) = 19.059, p = .000$$

In this example, the contrast alpha should be .025. Then, the high motivator group in high hygiene group is significant larger than low motivator, but, in low hygiene group, the high motivator is not significant larger than low motivator group.

Hygiene difference in each motivator group

Difference of hygiene in low motivator ($\bar{X}_{D\text{ in low motivator}} = 4$)

$$F(1,16) = 0.941, p = .346$$

Difference of hygiene in high motivator ($\bar{X}_{D\text{ in high motivator}} = 16$)

$$F(1,16) = 15.059, p = .001$$

In this example, the contrast alpha should be .025. Then, the high hygiene group in high motivator group is significant larger than low hygiene, but, in low motivator group, the high hygiene is not significant larger than low hygiene group.

Multiple Comparison in Main Effects

If null hypothesis in interaction effect is not rejected and one of the null hypotheses of the main effects is rejected, which population means in the rejected null hypothesis are not equal?

The group of procedure for comparing group means is multiple comparisons.

The general formula of null hypothesis of null hypothesis is

Null hypothesis	$H_0: \psi_{j.} = 0$	$H_0: \psi_{.k} = 0$	
Alternative hypothesis	$H_1: \psi_{j.} \neq 0$	$H_1: \psi_{.k} \neq 0$	(Two-tailed)
	$H_1: \psi_{j.} > 0; \psi_{j.} < 0$	$H_1: \psi_{.k} > 0; \psi_{.k} < 0$	(One-tailed)

This table shows rough classification of methods to compare multiple comparisons. (Like one-way ANOVA but the standard error in multiple comparisons formula is less than in one-way ANOVA)

	Homogeneity of variance		Heterogeneity of variance	
	Equal *n*	Unequal *n*	Equal *n*	Unequal *n*
Pairwise (Post hoc)	Tukey Bonferreni REGW-F	Tukey-Kramer Fisher-Hayter	Games-Howell	Games-Howell
Nonpairwise (Post hoc)	Scheffe	Scheffe	Brown-Forsythe	Brown-Forsythe

Example

Practical Significance

The eta squared in factorial design is the proportion of the effect that can be explained the total variance.

$$\eta_1^2 = \frac{SS_1}{SS_{\text{total}}}$$

$$\eta_2^2 = \frac{SS_2}{SS_{\text{total}}}$$

$$\eta_{1\times2}^2 = \frac{SS_{1\times2}}{SS_{\text{total}}}$$

The eta squared is similar to the squared partial correlation, pr^2, in regression analysis. However, the main effects and interaction effect are not collinear. Then, in balanced design (n in each cell are equal), the $pr^2 = r^2$.

The omega squared of desired effect that ignoring other effects is

$$\omega_1^2 = \frac{(s-1)(F_1-1)}{(s-1)(F_1-1)+n}$$

$$\omega_1^2 = \frac{(t-1)(F_2-1)}{(t-1)(F_2-1)+n}$$

$$\omega_{1\times2}^2 = \frac{(s-1)(t-1)(F_{1\times2}-1)}{(s-1)(t-1)(F_{1\times2}-1)+n}$$

Hedges' g statistic can be used to determine the effect size of contrasts among the diets.

$$g = \frac{|\hat{\psi}_i|}{\sqrt{\hat{\sigma}_{\text{Pooled}}}}$$

$$\hat{\sigma}^2_{\text{Pooled}} = \sqrt{MS_{\text{error}}}$$

Three-way Design

When there are three factors, the interaction effects will be the combination of these factors.

Main Effect	Interaction Effect
Factor 1	Factor 1 x Factor 2
Factor 2	Factor 1 x Factor 3
Factor 3	Factor 2 x Factor 3
	Factor 1 x Factor 2 x Factor 3

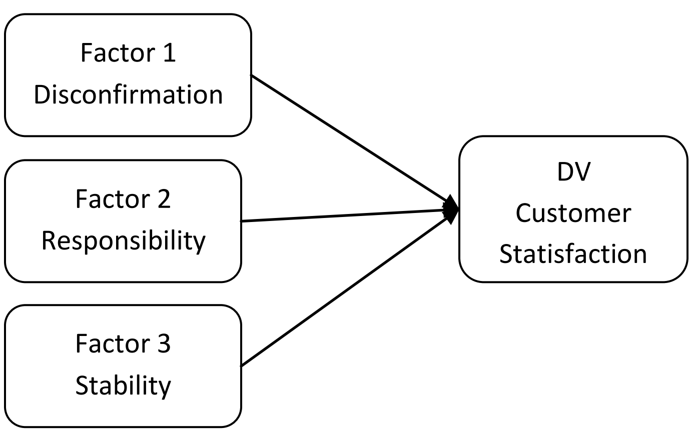
The total sum of squared deviation can be divided into

$$SS_{\text{total}} = SS_1 + SS_2 + SS_3 + SS_{1\times 2} + SS_{1\times 3} + SS_{2\times 3} + SS_{1\times 2\times 3} + SS_{\text{error}}$$

Therefore, the sample model equation is

$$X_{ijk} = \mu + \alpha_j + \beta_k + \gamma_l + (\alpha\beta)_{jk} + (\alpha\gamma)_{jl} + (\beta\gamma)_{kl} + (\alpha\beta\gamma)_{jkl} + \varepsilon_{i(jk)}$$

Example: Tsiros, Mittal & Ross (2004)



Factor 1: Positive/Negative

Factor 2: Company-related/
Company-unrelated

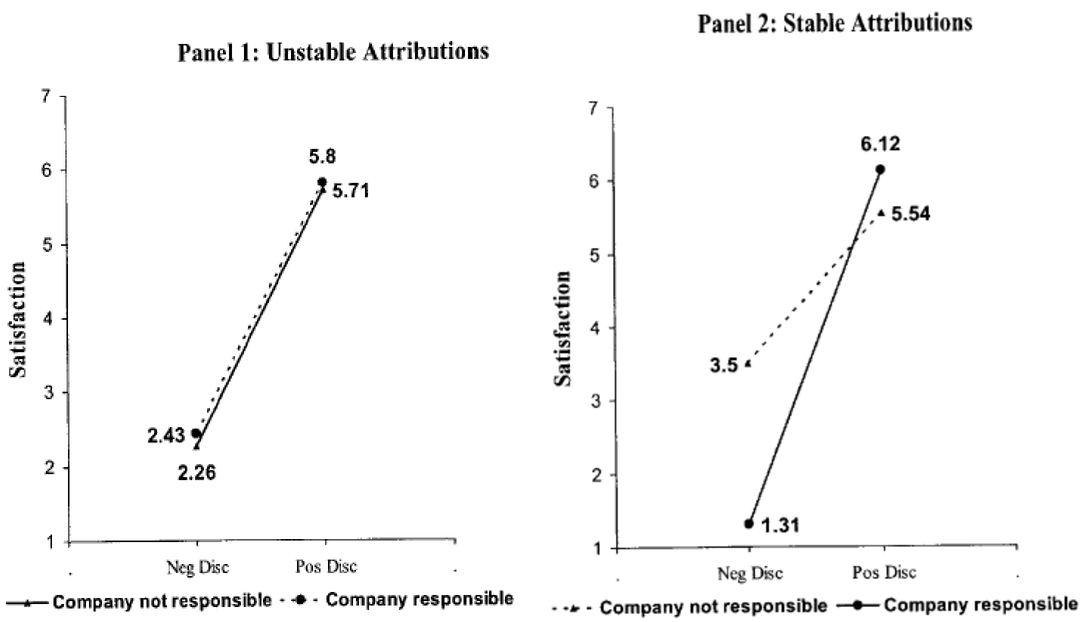
Factor 3: Stable/Unstable

DV: Customer Satisfaction (1-7)

The partition sources of variance and *F* test

Effect	*SS*	*df*	*MS*	*F*	*p*
Disconfirmation (D)	371.79	1	371.79	329.02	.001
Responsibility (R)	2.07	1	2.07	1.83	.178
Stability (S)	0.41	1	0.41	0.36	.550
D x R	20.85	1	20.85	18.45	.001
D x S	0.80	1	0.80	0.71	.405
R x S	2.26	1	2.26	2.00	.159
D x R x S	6.12	1	6.12	5.42	.020
Error	218.09	193	1.13		
Total	622.39	200			

When the 3-way interaction is significant, the good strategy to see interaction is plotting graph.



When the 3-way interaction occurs, the analysis of simple effect is sophisticated. It analyze whether the interaction between disconfirmation and responsibility on satisfaction in stable attribution is the same as in unstable attribution.