Normal Distribution and Introduction to Hypothesis Testing

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Why Normal Distribution is Important?

Population Distribution of Many Variables in Science and Nature

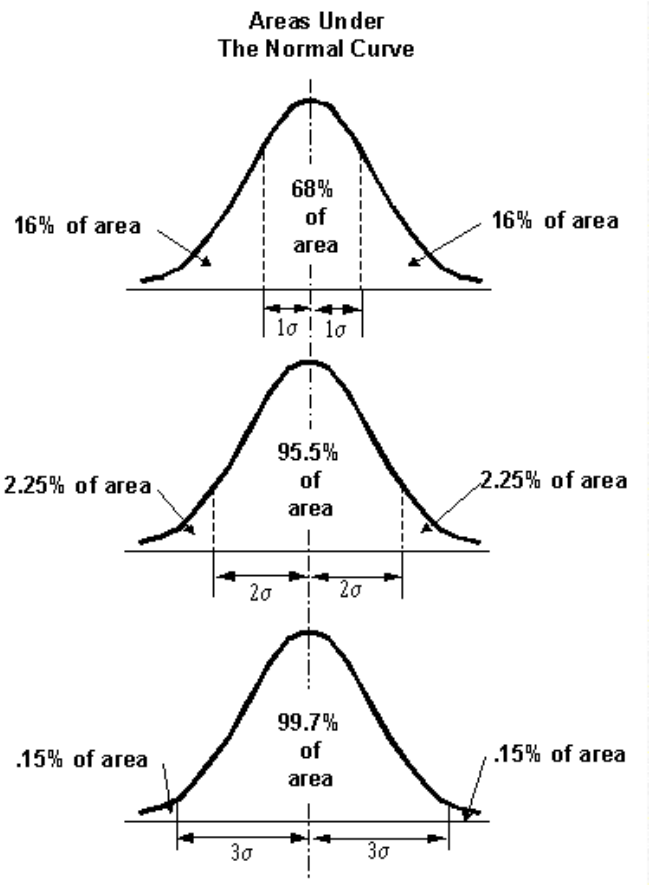
Distribution of Sample Statistic

Characteristics of Normal Distribution

Equation

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/(2\sigma^2)}$$

Graph of Normal Distribution



Author Note

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Standard Score

Linearly transformation from raw score to distribution that $\bar{X}$ = 0 and S = 1

This linearly transformed score is standard score (z score).

$$z = \frac{X - \bar{X}}{S}$$

Or

$$z = \frac{X - \mu}{\sigma}$$

If the shape of distribution is similar to normal distribution, you can use standard normal distribution table to find areas under it.

Area from the left to percentile point = percentile rank

Raw Score and Derived Score

Why raw score and percentage of score are useless?

Derived Score

- Percentile Rank
- Standard Score
- Transformed Standard Score

$$z' = S'z + \bar{X}'$$

Comparing Performance from different tests

Hypothesis Testing from Population Distribution

When you want to say that A is A, you have to know that what the characteristic of A is.

For example, when you say that this person is not female, you have to know what the characteristic of female is.

Research Hypothesis

Statistical Hypothesis

- Null Hypothesis
- Alternative Hypothesis

Example of Hypothesis Testing

Example 1

Female's Height in Thailand: $\mu = 155;\ \sigma = 10$

Male's Height in Thailand: $\mu = 170;\ \sigma = 10$

Research Hypothesis: A is the person who tall 151. A is not likely to be male.

Alternative Hypothesis: A is not male or $H_1: \mu < 170$

Null Hypothesis: A is male or $H_0: \mu \geq 170$

You must determine the value of height that is very unlikely to occur.

The acceptable risk of rejecting null hypothesis when it is true is 5 % or 1 % (making a decision error).

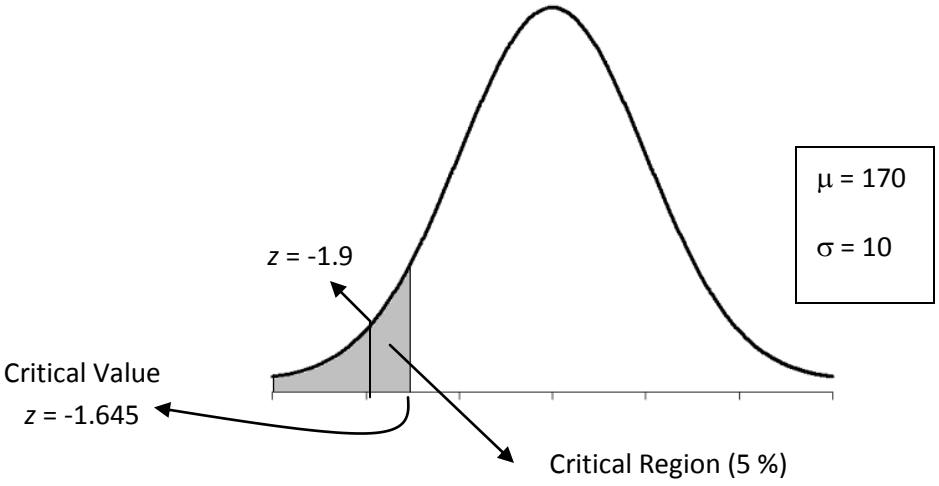
If the acceptable risk is 5 %, the standard score is 1.645.

Critical value/critical region

Compute: standard score of A height

$$z = \frac{X - \mu}{\sigma} = \frac{151 - 170}{10} = -1.9$$

Graph of Male Height Distribution



Standard score of A height fall in critical region, then the probability of A drawn from Thai male is less than 5 %, rejecting null hypothesis.

Example 2

IQ score in Thailand: $\mu = 100;\ \sigma = 15$

Research Question: You measure IQ from a child who is difficult to learn, he is received IQ = 80. Is he mental retard?

Research Hypothesis: He does not have normal intelligence.

Alternative Hypothesis: He is from the population of mental retarded person or $H_1: \mu < 100$

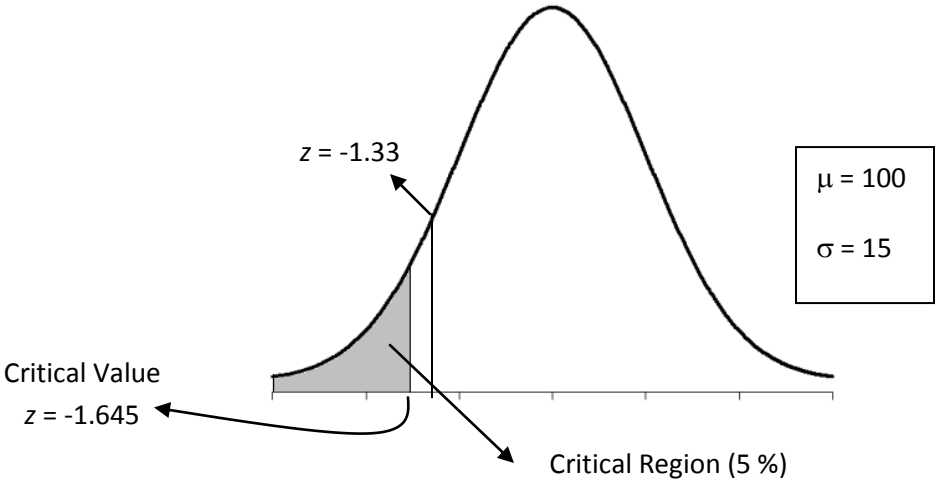
Null Hypothesis: He is from the population of normal intelligence or $H_0: \mu \geq 100$

If the acceptable risk is 5 %, the standard score is 1.645.

Compute: Standard score of his IQ

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 100}{15} = -1.33$$

Graph of Normal Intelligence People Distribution



Standard score of his IQ do not fall in critical value, then the probability of his IQ drawn from normal intelligence people is more than 5 %, fail to reject null hypothesis

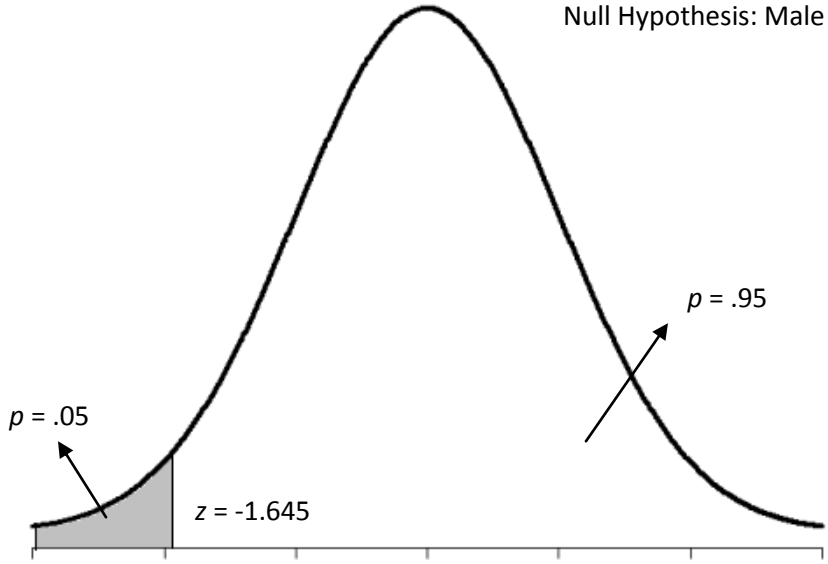
He may be normal intelligence person.

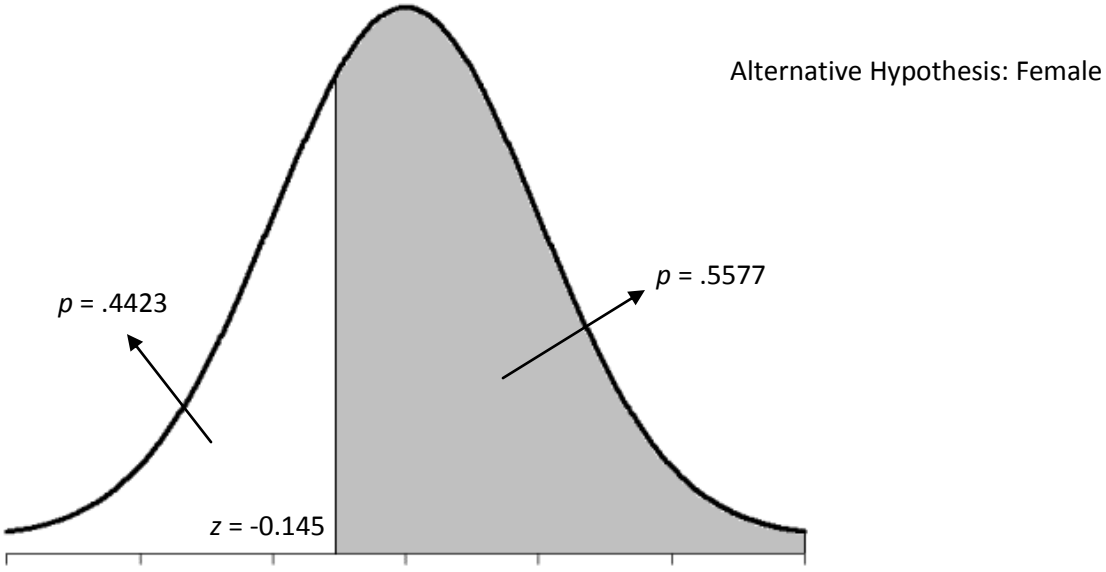
Error in Decision

Type I error (α)

Type II error (β)

Example I





The probability of type I error is 5 % (determined by researcher)

The probability of type II error is 55.77 %

The power of decision is 44.23 %

Decision Outcome

Researcher Decision	*True Situation*	
	H_0 True	H_0 False
Fail to reject H_0	Correct Acceptance	Type II error
Reject H_0	Type I error	Correct Rejection (Power)

Explanation when reject or fail to reject null hypothesis

1) Reject null hypothesis
    - Correct rejection
    - Reject true null hypothesis or type I error (Researcher can control this probability)
2) Fail to reject null hypothesis
    - Correct Acceptance
    - Fail to reject false hypothesis or type II error (Researcher cannot control)

Therefore, when fail to reject null hypothesis, you cannot say that null hypothesis is true.

Directional or one-side hypothesis

Nondirectional or two-side hypothesis

p value

Example of Hypothesis Testing Using *p* value

Example 1

Female's Height in Thailand: $\mu = 155;\ \sigma = 10$

Male's Height in Thailand: $\mu = 170;\ \sigma = 10$

Research Hypothesis: A is the person who tall 151. A is not likely to be male.

Alternative Hypothesis: A is not male or $H_1: \mu < 170$

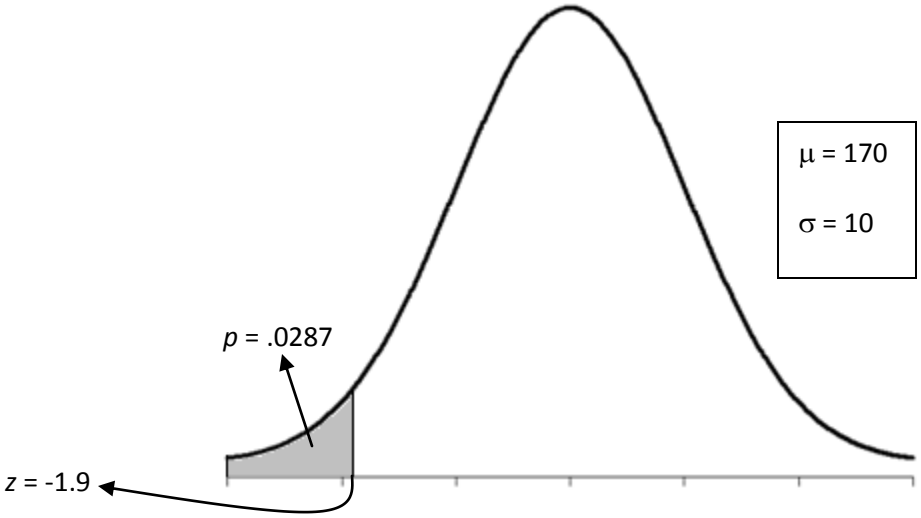
Null Hypothesis: A is male or $H_0: \mu \geq 170$

Compute: How much the probability that A is from the population of Thai male?

$$z = \frac{X - \mu}{\sigma} = \frac{151 - 170}{10} = -1.9$$

The probability that males tall 151 or lower is .0287 or 2.87 % (*p* = .0287)

Graph of Male Height Distribution



Is 2.87 % less enough of rejecting that null hypothesis is true?

If the acceptable risk of rejecting null hypothesis when it is true is 5 %, the null hypothesis is not tenable ($p < .05$).

If rejecting null hypothesis, then accept alternative hypothesis, that is, A is not male (may be female).

Example 2

IQ score in Thailand: $\mu = 100;\ \sigma = 15$

Research Question: You measure IQ from a child who is difficult to learn, he is received IQ = 80. Is he mental retard?

Research Hypothesis: He does not have normal intelligence.

Alternative Hypothesis: He is from the population of normal intelligence or $H_1: \mu < 100$

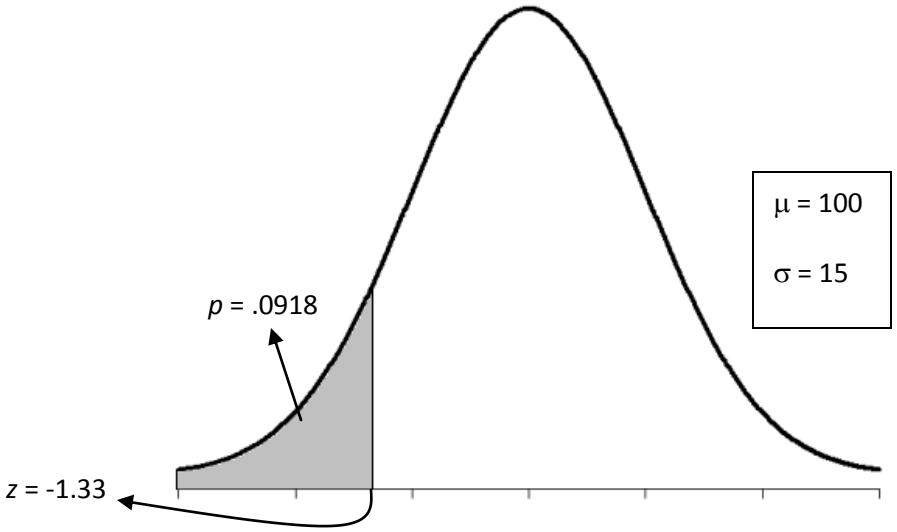
Null Hypothesis: He is from the population of mental retarded person or $H_0: \mu \geq 100$

Compute: How much the probability that he is from the population of normal intelligence?

$$z = \frac{X - \mu}{\sigma} = \frac{80 - 100}{15} = -1.33$$

The probability that normal intelligence people have IQ equal to 80 or lower is .0918 or 9.18 %.

Graph of Normal Intelligence People Distribution



If the acceptable risk of rejecting null hypothesis when it is true is 5 %, the null hypothesis is tenable ($p > .05$).

He may be normal intelligence person.

Steps for Hypothesis Testing