Interpretation of Multivariate Growth Curve Model

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I use the Model 27 from my R for Multilevel Models manual (which will be referred to as the manual) for an example in interpreting the multivariate growth curve model. Please see the manual to check how to fit the model. I will show you how to interpret each part of the output in this paper.

First, let's interpret the model fit information.

```
Linear mixed-effects model fit by maximum likelihood
 Data: long2
       AIC      BIC    logLik
  112866.7 113013.4 -56416.35
```

This part is the AIC and BIC of the model. You will use these values to compare with other models. Next, let's interpret the fixed effects:

```
Fixed effects: value ~ 0 + constparent + constparent:gradec + constpeer + constpeer:gradec
                      Value    Std.Error    DF    t-value p-value
constparent        2.0305157 0.015211684 35333 133.48395       0
constpeer          1.2778746 0.016440875 35333  77.72546       0
constparent:gradec -0.2220791 0.004016936 35333 -55.28569       0
gradec:constpeer   -0.1653800 0.004564085 35333 -36.23508       0
```

Note that each predictor represents:

- `constparent`: The parent encouragement values at Grade 1 of each student
- `constpeer`: The peer encouragement values at Grade 1 of each student
- `constparent:gradec`: The linear change in parent encouragement (over grades) of each student
- `gradec:constpeer`: The linear change in peer encouragement (over grades) of each student.

Therefore, the fixed effects can be interpreted as follows:

- The average (across students) parent encouragement values at Grade 1 was 2.03, which was significant, $t(35333) = 133.48$, $p < .001$. This value is γ_{00} in the manual.
- The average peer encouragement values at Grade 1 was 1.28, which was significant, $t(35333) = 77.73$, $p < .001$. This value is γ_{20} in the manual.
- The parent encouragement scores significantly changed by -0.22 (as grade increases by 1) on average (across students), $t(35333) = -55.25$, $p < .001$. On average, the scores decreased over time. This value is γ_{10} in the manual.
- The peer encouragement scores significantly changed by -0.16 on average, $t(35333) = -36.24$, $p < .001$. On average, the scores decreased over time. This value is γ_{30} in the manual.

Next, let's interpret the random effects part.

```
Random effects:
 Formula: ~0 + constparent + constparent:gradec + constpeer + constpeer:gradec | caseid
 Structure: General positive-definite, Log-Cholesky parametrization
              StdDev    Corr
```

```
constparent        0.7069079 cnstprn constpr cnstp:
constpeer          0.6833550  0.376
constparent:gradec 0.1086271 -0.528  -0.271
gradec:constpeer   0.1329360 -0.187  -0.701   0.265
Residual           0.7594011
```

This part provides you the standard deviation and correlation between random effects.

Let's see the `StdDev` column. This function represents the standard deviation of the random effects across students (L2 units). That is,

- The standard deviation of the parent encouragement values at Grade 1 across students was 0.71. This value is $\sqrt{\tau_{00}}$ in the manual.
- The standard deviation of the peer encouragement values at Grade 1 across students was 0.68. 0.71. This value is $\sqrt{\tau_{22}}$ in the manual.
- The standard deviation of the linear changes in parent encouragement across students was 0.11. 0.71. This value is $\sqrt{\tau_{11}}$ in the manual.
- The standard deviation of the linear changes in peer encouragement across students was 0.13. 0.71. This value is $\sqrt{\tau_{33}}$ in the manual.

The residual standard deviation at the measurement level will be interpreted later because the residual standard deviations were different across dependent variables.

Readers may make expected ranges of the initial statuses and changes of students by the formula similar to the confidence interval formula:

$$95\% \text{ Expected Range} = \hat{\gamma}_{ii} \pm \left(1.96 \times \sqrt{\tau_{ii}}\right)$$

Therefore, 95% of students will have the following values in following ranges:

- The parent encouragement values at Grade 1: $2.03 \pm (1.96 \times 0.71) = (0.64, 3.42)$
- The peer encouragement values at Grade 1: $1.28 \pm (1.96 \times 0.68) = (-0.06, 2.62)$
- The linear change in parent encouragement: $-0.22 \pm (1.96 \times 0.11) = (-0.43, -0.01)$
- The linear change in peer encouragement: $-0.17 \pm (1.96 \times 0.13) = (-0.43, 0.10)$

For example, 95% of students had the change in parent encouragement from -0.43 to 0.10. You may notice that parent encouragement decreased for almost all students; however, peer encouragement decreased for most students but increased for some students.

Let's see the `Corr` part. This output represents the correlation between the random effects. This output may be not easy to understand for some people. Furthermore, sometimes, the order of random effects is not the way we want it. I recommend making your own correlation matrix from the output. First, make the order of the correlation matrix similar to the output.

	Parent Grade 1	Peer Grade 1	Change Parent	Change Peer
Parent Grade 1	1			
Peer Grade 1	.376	1		
Change Parent	-.528	-.271	1	
Change Peer	-.187	-.701	.265	1

Because we have four random effects, we will use 4 by 4 matrix. The diagonal elements are always 1. The off-diagonal elements are those values listed in the `Corr` section. Please make sure to label your rows and columns in the way that make sense for you.

Next, reorder the order of variables in the way that you want. For example, I want to order it as parent grade 1, change parent, peer grade 1, change peer. Thus, my new correlation matrix would be

	Parent Grade 1	Change Parent	Peer Grade 1	Change Peer
Parent Grade 1	1			
Change Parent	-.528	1		
Peer Grade 1	.376	-.271	1	
Change Peer	-.187	.265	-.701	1

These values are the following notations in the R manual:

	β_{0j}	β_{1j}	β_{2j}	β_{3j}
β_{0j}	1			
β_{1j}	$\rho_{10} = \dfrac{\tau_{10}}{\sqrt{\tau_{11}\tau_{00}}}$	1		
β_{2j}	$\rho_{20} = \dfrac{\tau_{20}}{\sqrt{\tau_{22}\tau_{00}}}$	$\rho_{21} = \dfrac{\tau_{21}}{\sqrt{\tau_{22}\tau_{11}}}$	1	
β_{3j}	$\rho_{30} = \dfrac{\tau_{30}}{\sqrt{\tau_{33}\tau_{00}}}$	$\rho_{31} = \dfrac{\tau_{31}}{\sqrt{\tau_{33}\tau_{11}}}$	$\rho_{32} = \dfrac{\tau_{32}}{\sqrt{\tau_{33}\tau_{22}}}$	1

Then, interpret all correlations:

- The correlation between the parent encouragement values at Grade 1 and the changes in parent encouragement was -.528. This value is ρ_{10} in the manual.
- The correlation between the parent encouragement values at Grade 1 and the peer encouragement values at Grade 1 was .376. This value is ρ_{20} in the manual.
- The correlation between the parent encouragement values at Grade 1 and the changes in peer encouragement was -.187. This value is ρ_{30} in the manual.
- The correlation between the changes in parent encouragement and the peer encouragement values at Grade 1 was -.271. This value is ρ_{21} in the manual.
- The correlation between the changes in parent encouragement and the change in peer encouragement was .265. This value is ρ_{31} in the manual.
- The correlation between the peer encouragement values at Grade 1 and the changes in peer encouragement was -.701. This value is ρ_{32} in the manual.

For both dependent variables, if their values were high at Grade 1, the changes were more negative (had a stronger drop in their values; -.528 and -.701 for parent and peer respectively). Both dependent variables were slightly positively correlated at Grade 1 (.376) and their changes were slightly positive (.265).

Next, let's interpret the residual standard deviation:

```
Variance function:
 Structure: Different standard deviations per stratum
 Formula: ~1 | variable
 Parameter estimates:
parentpush   peerpush
  1.000000   1.168075
```

The residual standard deviations provided in the Random effects part was 0.76. This value can be used to calculate the residual standard deviations of each dependent variable. We will use the values provided from the `Variance function` part and multiplied them with the residual standard deviations of 0.76 above:

- The residual variance of parent encouragement at the measurement level after accounting for grades was $0.76 \times 1.00 = 0.76$. This value matches with $\sqrt{\sigma_A^2}$ in the manual.
- The residual variance of peer encouragement at the measurement level after accounting for grades was $0.76 \times 1.17 = 0.89$. This value matches with $\sqrt{\sigma_B^2}$ in the manual.

Next, let's interpret the correlation between residuals at measurement level (Level 1):

```
Correlation Structure: General
 Formula: ~1 | caseid/obs
 Parameter estimate(s):
 Correlation:
  1
2 0.12
```

In this case, the correlation between the residuals of parent and peer encouragements at measurement level after accounting for grades was 0.12. This value matches with $\rho_{BA}^{(1)} = \sigma_{BA}/\sqrt{\sigma_A^2 \sigma_B^2}$ in the manual.

The sample sizes of the data can be obtained from the output:

```
Number of Observations: 41280
Number of Groups: 5944
```

The number of rows was 41,280. The number of students was 5,944. Thus, there are 6.94 rows for each student. Note that each measurement has two rows (for two dependent variables) so the number of measurements for each student was 3.47.

I recommend not interpreting the following parts for now:

```
Correlation:
                 cnstprn constpr cnstp:
constpeer          0.236
constparent:gradec -0.762  -0.165
gradec:constpeer   -0.150  -0.825   0.167

Standardized Within-Group Residuals:
      Min         Q1        Med         Q3        Max
-2.7390889 -0.6244660 -0.1355851  0.5876161  3.8055511
```