Assignment 2: Simulation in R (Summer 15)

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In class 4, simple mediation model is introduced. The main goal of the mediation model is to check whether mediator explains the effect of Y on X. The indirect effect, $\beta_{YM} \cdot \beta_{MX}$, is tested. However, the indirect effect statistic is not standardized. Analysts are not sure whether the size of indirect effect is high or low. Therefore, effect size statistics for indirect effect are needed. In this assignment, I would like to introduce two effect size statistics for indirect effect [Preacher and Kelley, 2011]. First, the ratio of the indirect effect to the direct effect (R_M) can be calculated by

$$R_M = \frac{\beta_{MX} \cdot \beta_{YM}}{\beta_{YX}}. \tag{1}$$

Second, completely standardized indirect effect (θ) can be calculated by:

$$\theta = \beta_{MX} \cdot \beta_{YM} \cdot \frac{\sigma_X}{\sigma_Y}. \tag{2}$$

where σ_X and σ_Y are total variance of X and Y.

Please conduct a simulation to show that the performance of the estimates of these effect statistics. That is, these statistics are checked whether they provide biased estimates of the population values. The design conditions should vary both the magnitude of indirect effects, the magnitude of total effects, and sample sizes.

Note that the variance of Y reported in Mplus is residual variances. You may derive the total variance of Y by the following equations:

$$\begin{aligned}
Y &= \alpha_Y + \beta_{YX} M + \beta_{YX} X + \epsilon_Y \\
&= \alpha_Y + \beta_{YX} (\alpha_M + \beta_{MX} X + \epsilon_M) + \beta_{YX} X + \epsilon_Y \\
&= \alpha_Y + \beta_{YX} \alpha_M + (\beta_{YX} \beta_{MX} + \beta_{YX}) X + \beta_{YX} \epsilon_M + \epsilon_Y \\
\text{Var}(Y) &= \text{Var}\left(\alpha_Y + \beta_{YX} \alpha_M + (\beta_{YX} \beta_{MX} + \beta_{YX}) X + \beta_{YX} \epsilon_M + \epsilon_Y\right) \\
&= \text{Var}\left((\beta_{YX} \beta_{MX} + \beta_{YX}) X + \beta_{YX} \epsilon_M + \epsilon_Y\right) \\
&= \text{Var}\left((\beta_{YX} \beta_{MX} + \beta_{YX}) X\right) + \text{Var}\left(\beta_{YX} \epsilon_M\right) + \text{Var}\left(\epsilon_Y\right) \\
&= (\beta_{YX} \beta_{MX} + \beta_{YX})^2 \text{Var}\left(X\right) + \beta_{YX}^2 \text{Var}\left(\epsilon_M\right) + \text{Var}\left(\epsilon_Y\right)
\end{aligned}$$

Hint:
You need to make sure that variance of X is estimated in the model. Estimate total variance of Y and two effect size statistics using `MODEL CONSTRAINT:` command.

References

Kristopher J Preacher and Ken Kelley. Effect size measures for mediation models: Quantitative strategies for communicating indirect effects. *Psychological Methods*, 16(2):93–115, 2011.